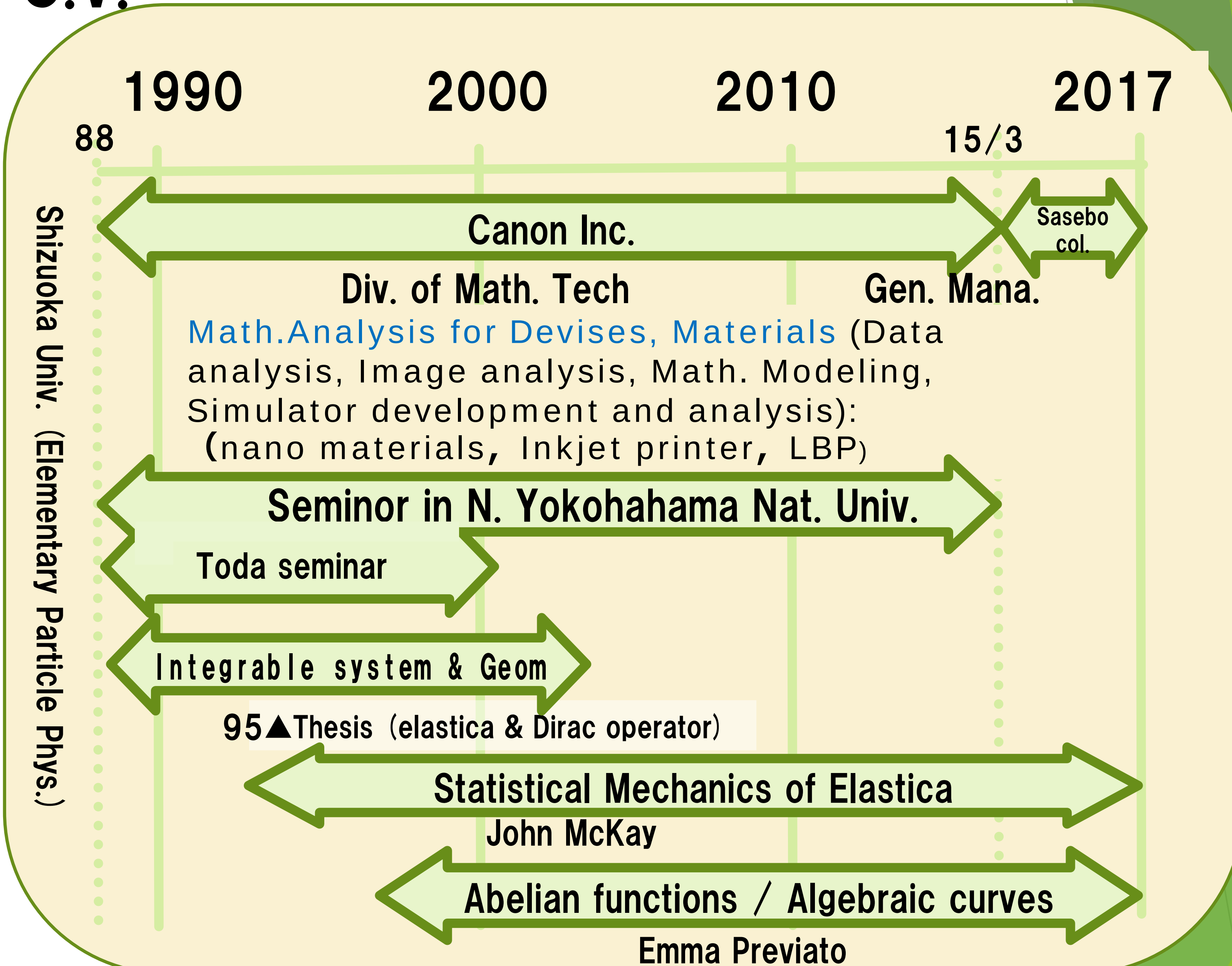
Advanced mathematical investigation for conductivity of highly disordered carbon systems associated with percolation and graph zeta function

The 9th MathAM-OIL Seminar
Sendai, Tohoku
Oct.6. 2017

Joint work with Iwao Sato, Akira Suzuki
Yoshiyuki Shimosako and Yunhong.Wang

Shigeki Matsutani
National institution of technology, Sasebo college,
IMI of Kyushu Univ.

C.V.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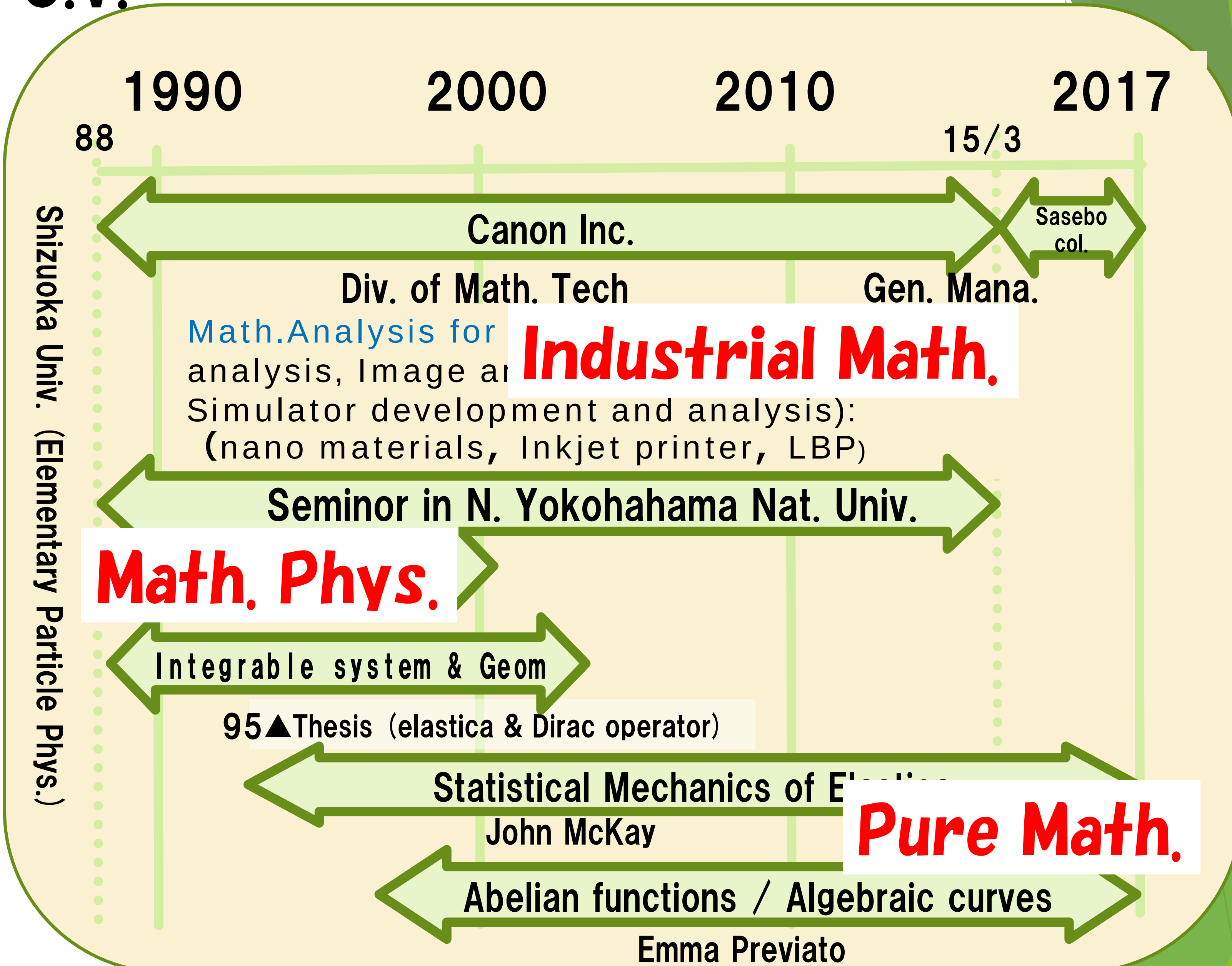
15/3
Shizuoka Univ. (Elementary Particle Phys.)
Canon Inc.
Sasebo col.
Div. of Math. Tech
Gen. Mana.
Math.Analysis for Devises, Materials (Data analysis, Image analysis, Math. Modeling, Simulator development and analysis):
(nano materials, Inkjet printer, LBP)
Seminor in N. Yokohahama Nat. Univ.
Toda seminar
Integrable system & Geom
95▲Thesis (elastica & Dirac operator)
Statistical Mechanics of Elastica
John McKay
Abelian functions / Algebraic curves
Emma Previato

C.V.



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Statistical Mechanics of E
John McKay
Pure Math.
Abelian functions / Algebraic curves
Emma Previato

Electric Devices

1. M. Okuda, Shigeki Matsutani,A. Asai, A. Yamano, K. Hatanaka, T. Hara and T. Nakagiri, *Electron trajectory analysis of surface condction electron emitter displays(SEDs)* (invited talk), SID 98 Digest, (1998) 185-188
2. S. Matsutani, M. Okuda and A. Asai, *Dynamics of electrons in half-space with cylindrical electro-static field*, Jpn J. Ind. Appl. Math., **18** (2001) 777-790,

CFD(Comp. Fluid Dynamics)

1. S. Matsutani, K. Nakano, and K. Shinjo, *Surface tension of multi-phase flow with multiple junctions governed by the variational principle*, Math. Phys. Analy. Geom. **14** (3) (2011) 237-278
2. Shigeki Matsutani, *Sheaf-theoretic investigation of CIP-method*, Appl. Math. Comp. **217** (2) (2010) 568-579

Nano Material

1. S. Matsutani, Y. Shimosako and Y.Wang, *Fractal Structure of Equipotential Curves on a Continuum Percolation Model* Physica A **391** (23) (2012) 5802-5809, Dec. 1, 2012, arXiv:1107.2983.
2. S. Matsutani, Y. Shimosako and Y. Wang, *Numerical Computations of Conductivities over Agglomerated Continuum Percolation Models*,

#total published papers =80(since 1991)

Math. Phys.

1. S Matsutani and H Tsuru, Reflectionless quantum wire, J. Phys. Soc. Jpn., 60 (11) (1991) 3640-3644, Nov. 15, 1991.
2. S. Matsutani and H. Tsuru, Physical relation between quantum mechanics and soliton on a thin elastic rod, Phys. Rev. A, 46 (1992) 1144-1147.
3. S. Matsutani, Quantum field theory on curved low dimensional space embedded in three dimensional space Phys. Rev. A, 47 (1993) 686-689,.
4. S. Matsutani, The Physical meaning of the embedded effect in the quantum submanifold system, J. Phys. A: Math. & Gen., 26 (19) (1993) 5133-5143.
5. S. Matsutani, Anomaly on a submanifold system: new index thoerem related to a submanifold system, J. Phys. A: Math. & Gen., 28 (5) (1995) 1399-1412.
6. S. Matsutani and A. Suzuki, Hopping conductivity associated with activation energy in disordered carbon, Phys. Lett A, 216 (1-5) (1996) 178-182.
7. S. Matsutani, A constant mean curvature surface and the Dirac operator, J. Phys. A: Math. & Gen., 30 (11) (1997) 4019-4029.
8. S. Matsutani, Statistical mechanics of elastica on a plane: origin of the MKdV hierarchy, J. Phys. A: Math. & Gen., 31 (11) (1998) 2705-2725.
9. S. Matsutani, On density of state of quantized Willmore surface :a way to a quantized extrinsic string in R^3, J. Phys. A, 31 (1998) 3595-3606.
10. S. Matsutani and Akira Suzuki, Apparent metal-insulator transition in disordered carbon, Phys. Rev. B, 62 (21) (2000) 13812-13815.

Algebraic Curves & Abelian Functions

1. S. Matsutani, *Hyperelliptic loop solitons with genus g: investigation of a quantized elastica*, , J. Geom. Phys., , **43** (2002) 146-162,

2. J.C. Eilbeck, V.Z. Enol'skii, S. Matsutani, Y. O^ nishi, and E. Previato, *Addition formulae over the Jacobian pre-image of hyperelliptic Wirtinger varieties*, Journal four die reine und angewandte Mathematik (Crelles Journal), (2008) **2008** No. 619 37-48

3. S. Matsutani, E. Previato A generalized Kiepert formula for *Cab* curves, Israel J. Math., **171** (2009) 305-323,

4. S.Matsutani E. Previato, *Jacobi inversion on strata of the Jacobian of the Crs curve yr = f (x), II*, J. Math. Soc. Japan, **60** (2008) 1009-1044, **66** (2014) 647-691,

5. J. Komeda, S. Matsutani and E. Previato, *The Riemann constant for a non-symmetric Weierstrass semigroup*, Archiv der Mathematik 2016

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Algebraic Curves & Abelian Functions

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I gave an invited talk at Algebraic Geometry Session in Math. Society Conference Sept 14, 2017 Yamagata.

J. Math. Soc. Japan, **60** (2008) 1009-1044, **66** (2014) 647-691,

5. J. Komeda, S. Matsutani and E. Previato, *The Riemann constant for a non-symmetric Weierstrass semigroup*, Archiv der Mathematik 2016

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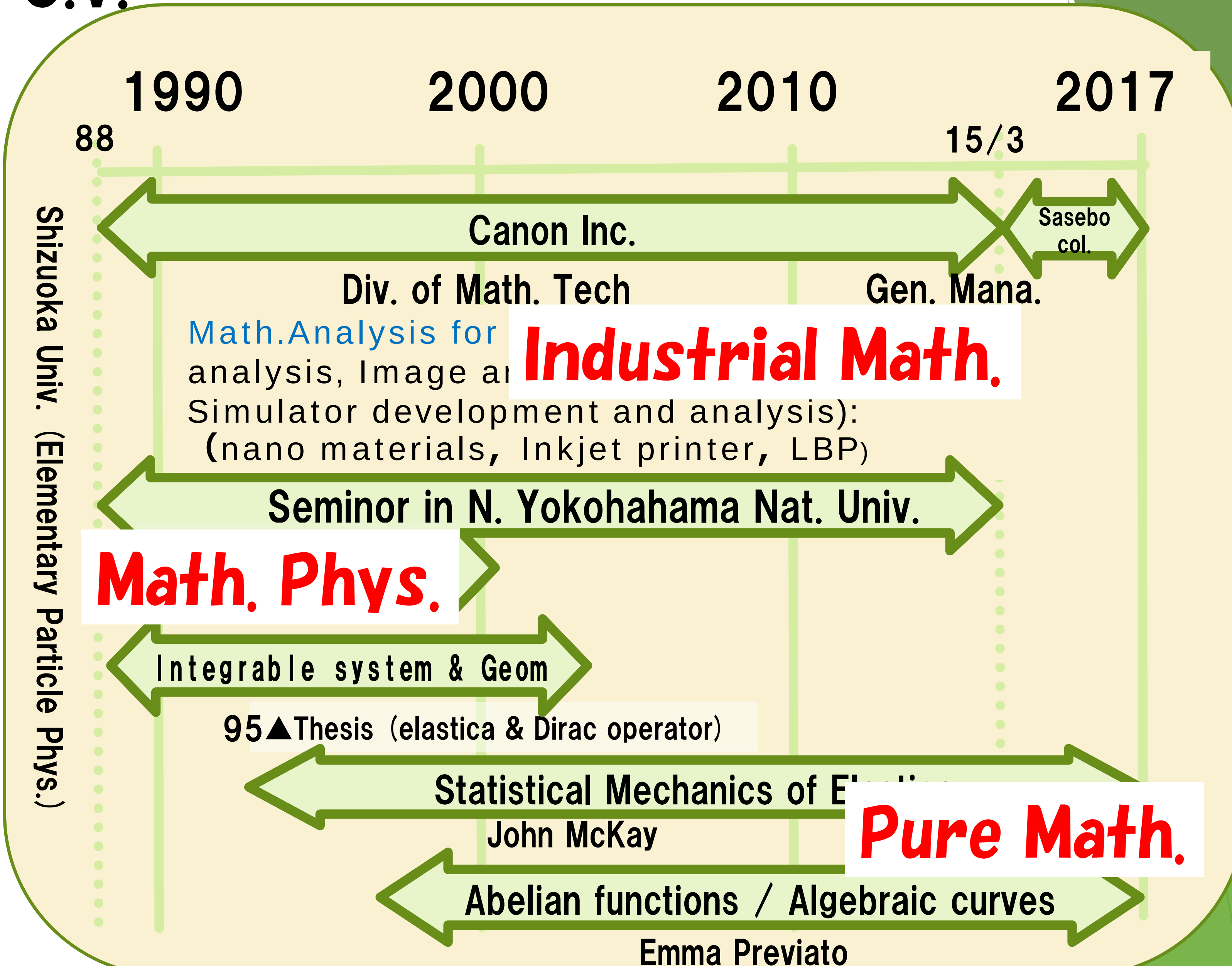

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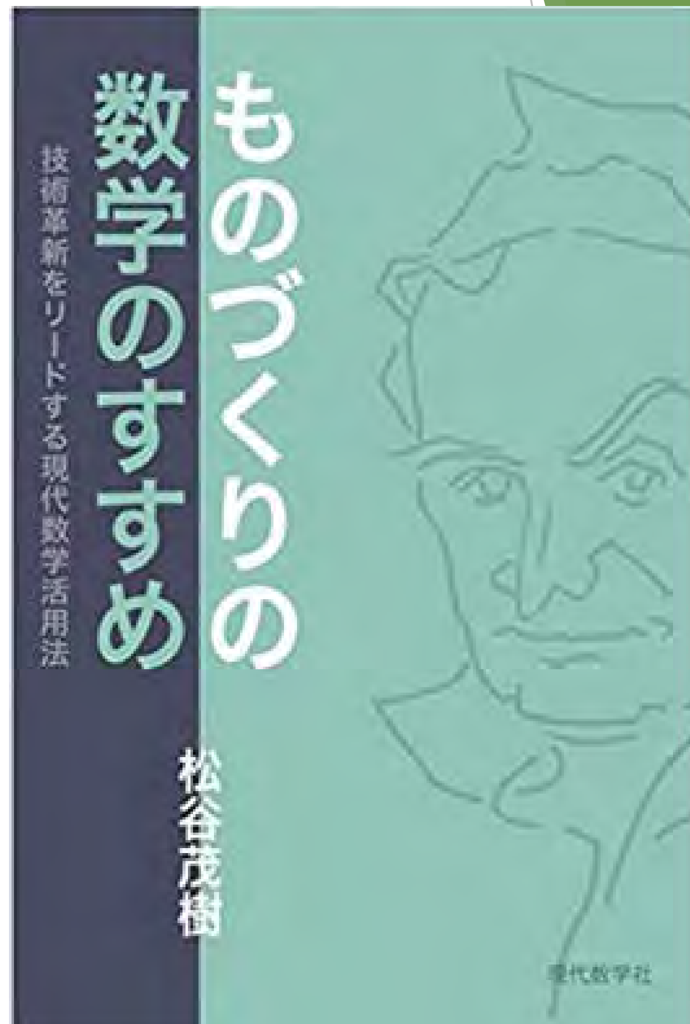
Abelian functions / Algebraic curves
Emma Previato
Shizuoka Univ. (Elementary Particle Phys.)

線型代数周遊ー応用をめざして



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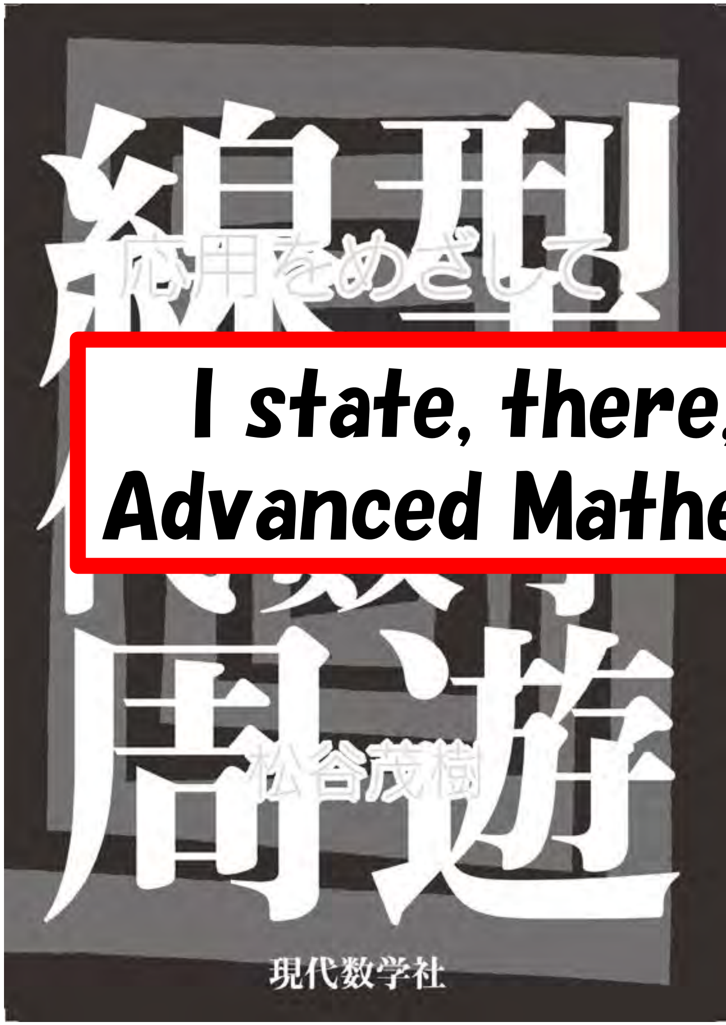
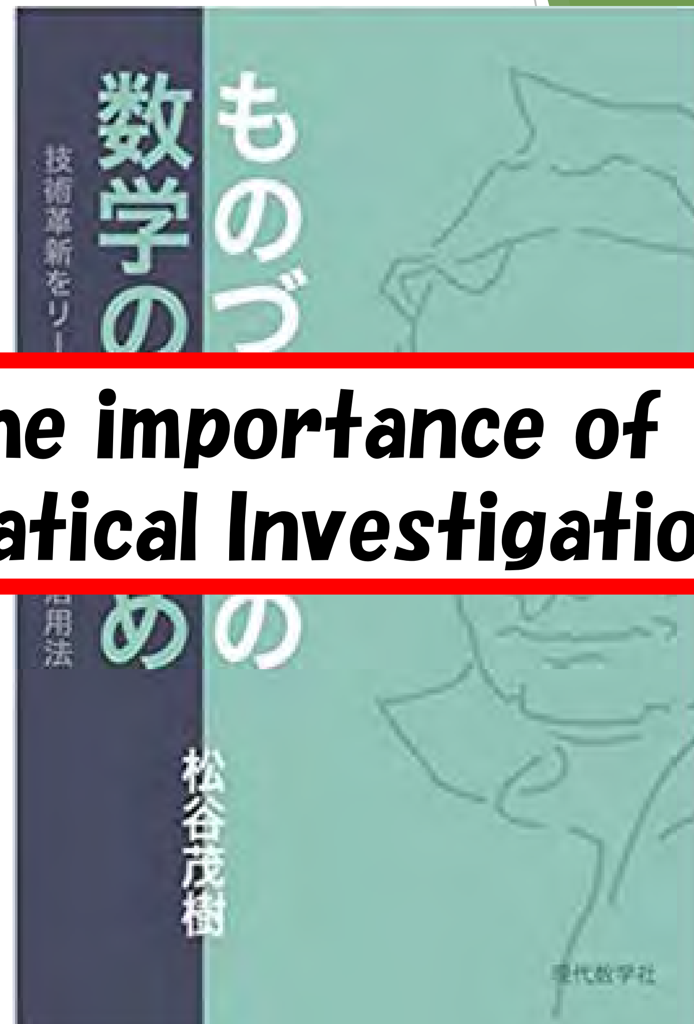
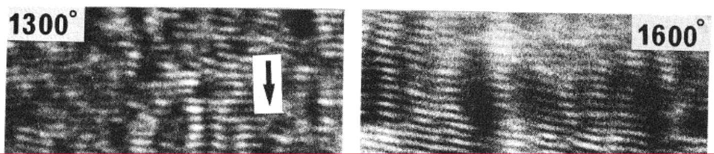




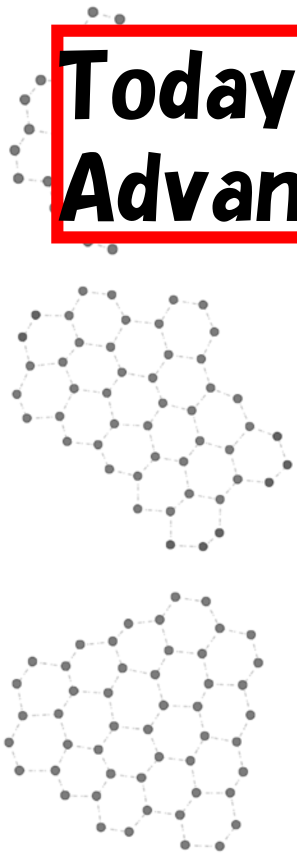
I state, there, the importance of Advanced Mathematical Investigation

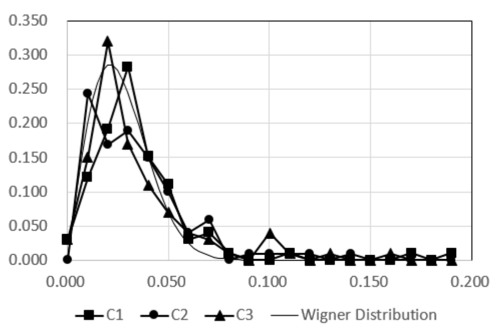
Adv. Math. Inves. on Conductivity of Disordered Carbon

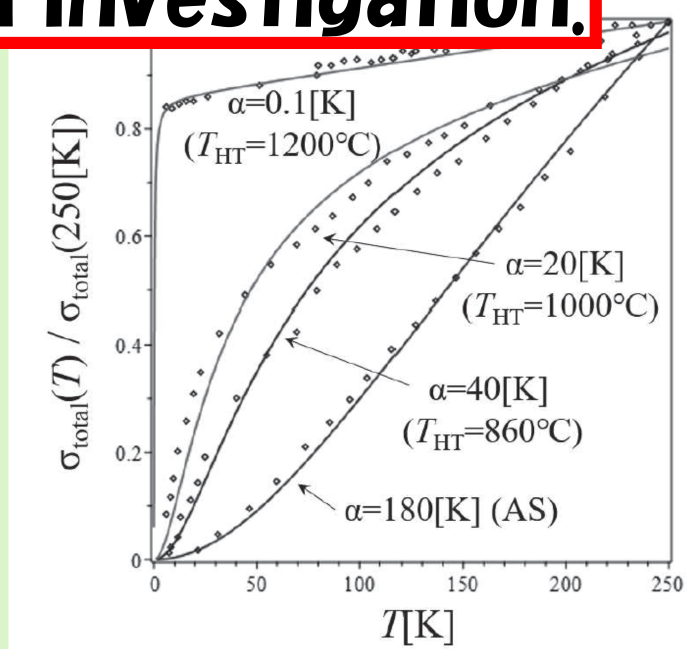
graph ζ function, Random Matrix theory Percolation theory
Phys.Lett. A (2017M-Sato)



Today's topic is one of examples of Advanced Mathematical Investigation.







What is Advanced Mathematical Investigation?

Crucial Industrial problems basically cannot be solved in the single field of mathematics or in the single field in science in general.

The Advanced Mathematical Investigation is an investigation on novel devices or materials from more practical points of view by using results of various math fields, including pure math, applied math and so on,

What is Advanced Mathematical Investigation?

In order to design something, we have to represent parameter dependence of its behavior.

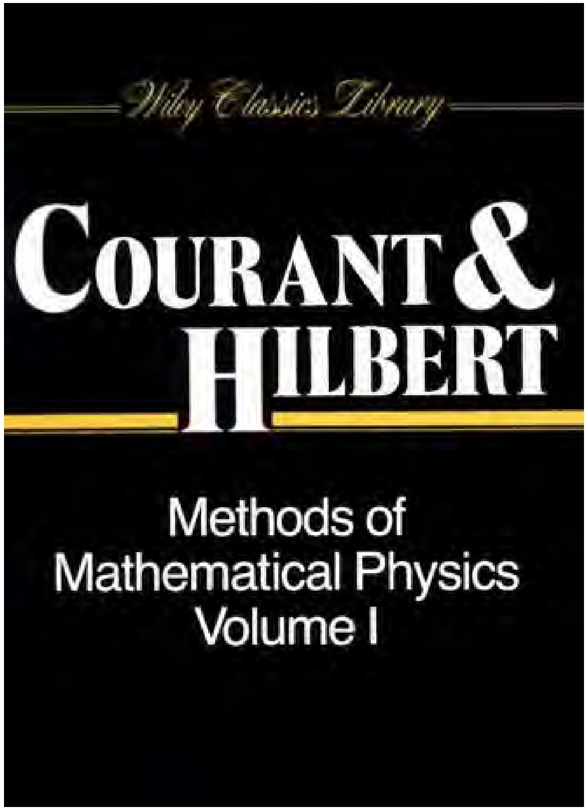
Its style in industrial mathematics might differ from the academic style, a little bit, because in order to represent the behavior, we use whatever we can.

What is Advanced Mathematical Investigation?

In order to design something, we have to represent parameter dependence of its behavior.

Its style in industrial mathematics might differ from the academic style, a little bit, because in order to represent the behavior, we use whatever we can.

But it is important that we use the various but so primitive tools.
It should not be exaggerated: without the tools, we cannot represent these phenomena.



The book "Courant-Hilbert" has played an important role to give mathematics for industry and physics, even now

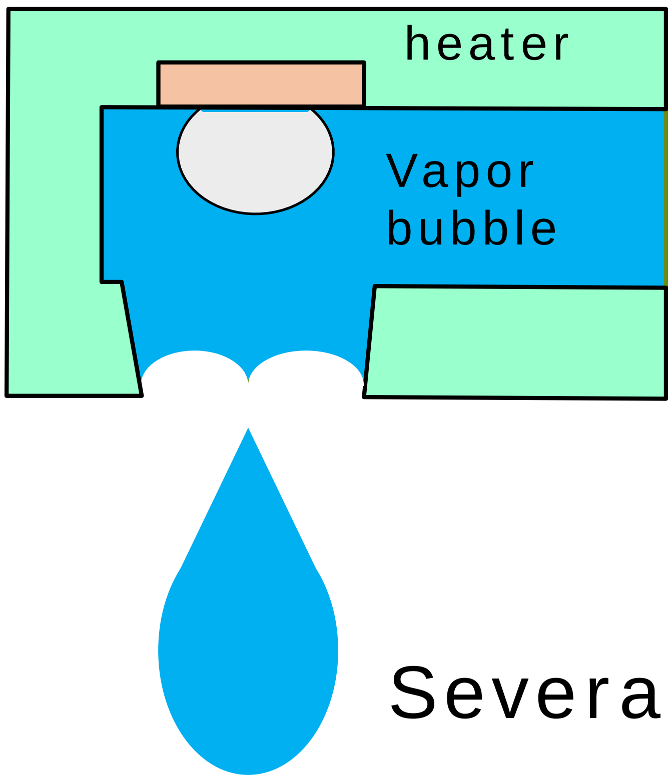
But about a century passed since it was published!

We need more mathematical tools because technology is developed in the century, though C-H goes on to plays the important role.

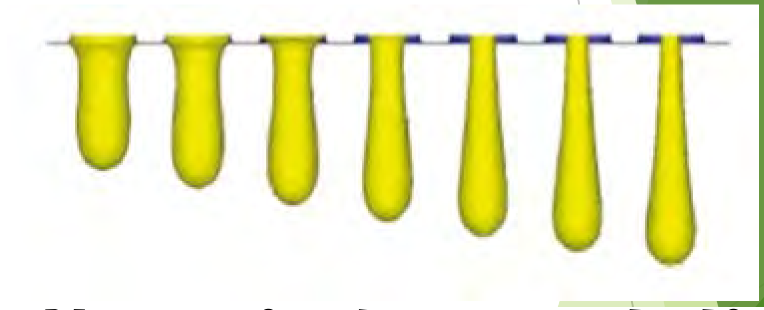
Mathematical Modeling of CFD

Problem: Modelize the three-phase fluid dynamics with triple-junction to predict the behavior of ink-jet numerically and to design an ink jet head!

Emission device of ink droplets

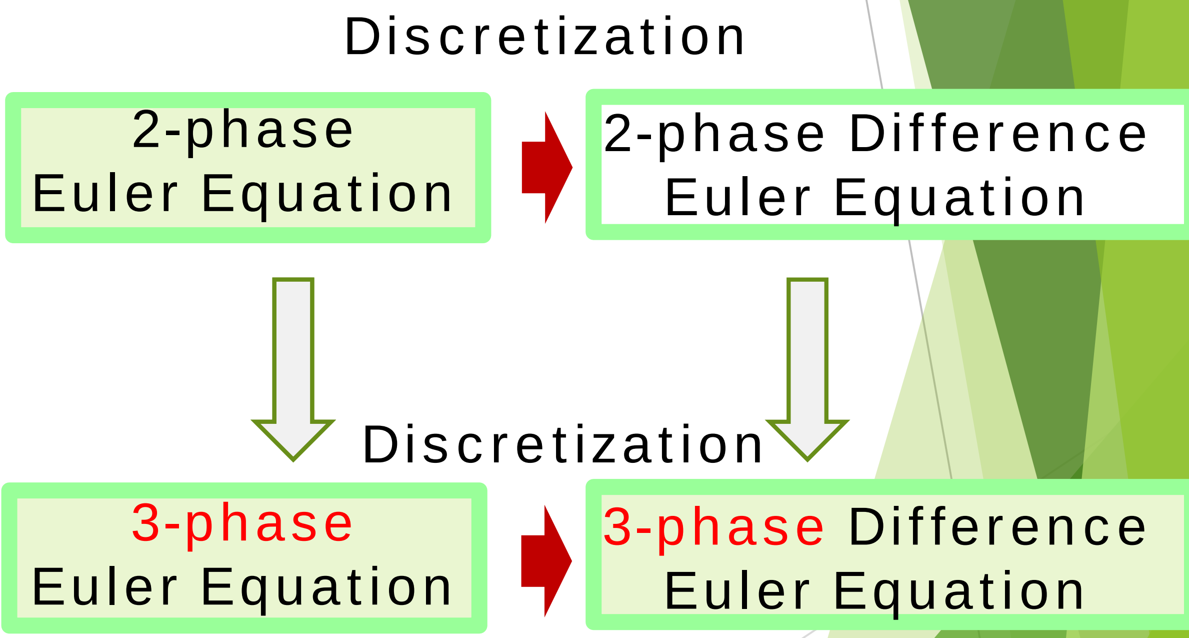


Several x 10 um

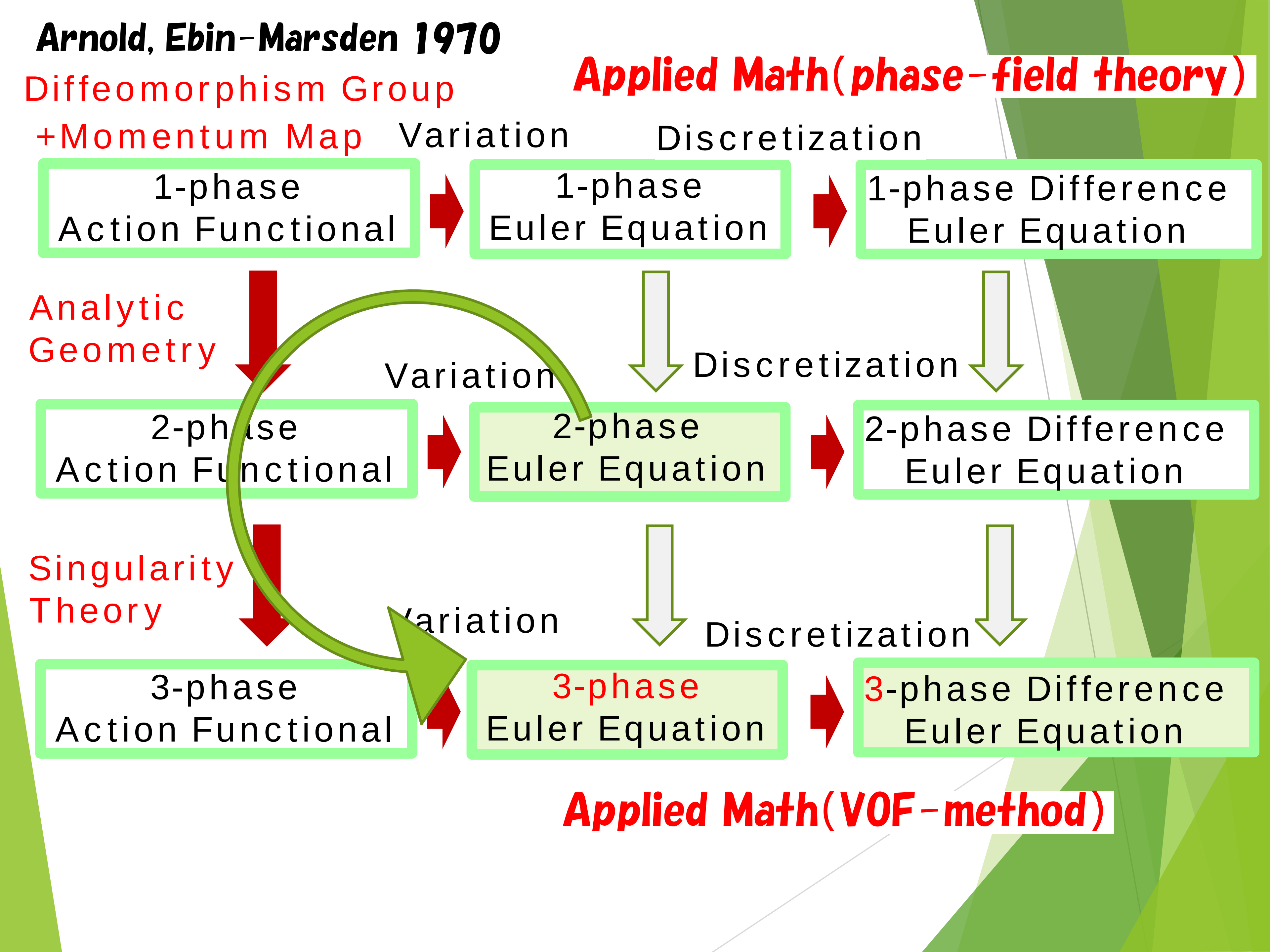


Numerical computation:
emission of ink droplets

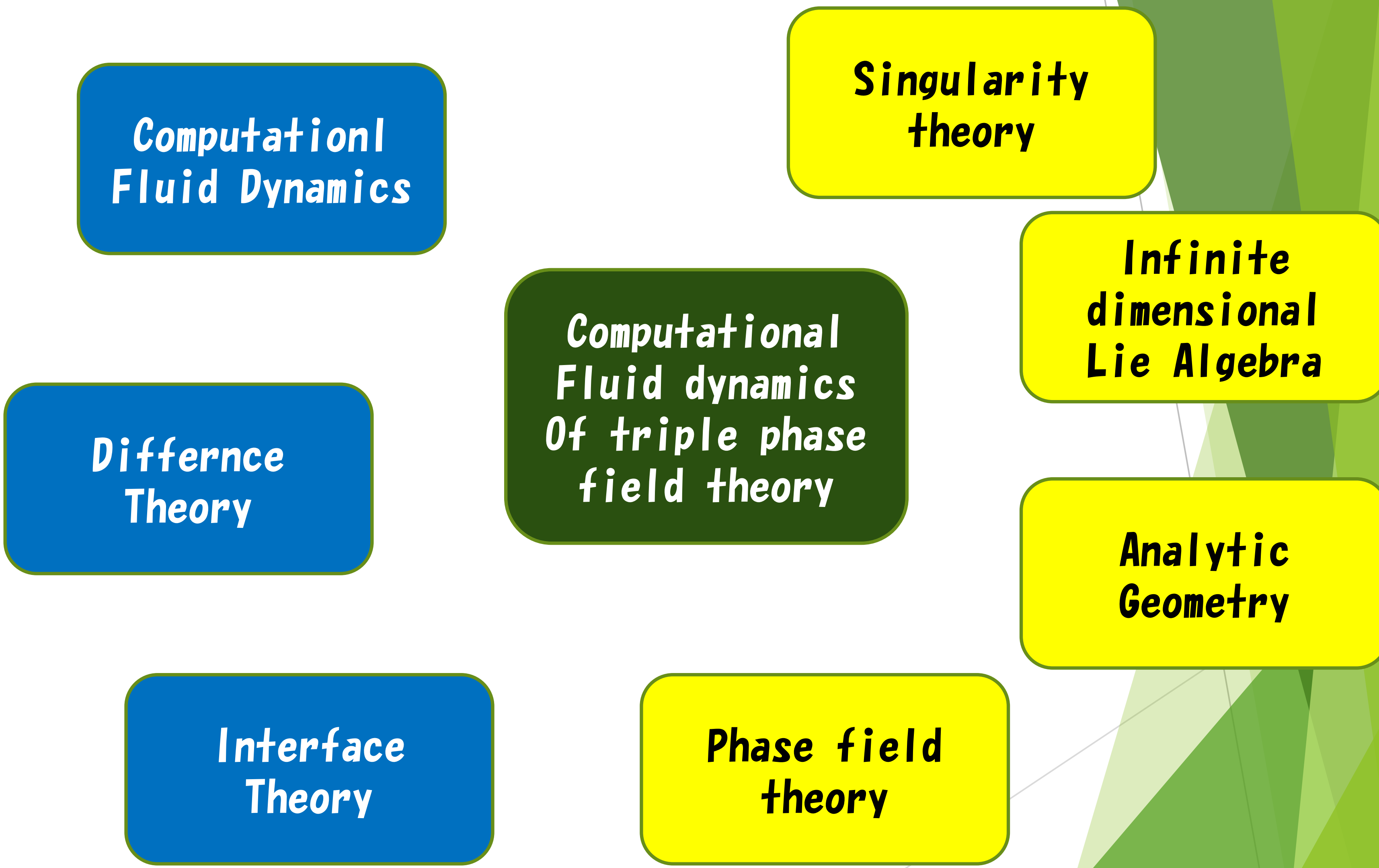
By revising the mathematical model of two-phase fluid dynamics, we obtained that of three-phase field!



Applied Math(VOF-method)

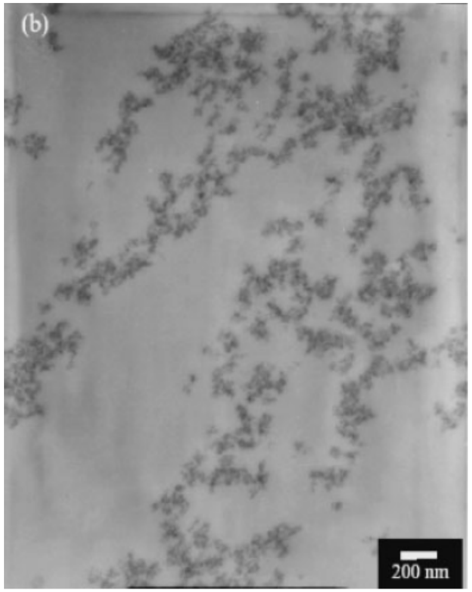

Arnold, Ebin-Marsden 1970
Diffeomorphism Group
+Momentum Map
Applied Math(phase-field theory)
Variation
Discretization
1-phase Action Functional
1-phase Euler Equation
1-phase Difference Euler Equation
Analytic Geometry
Variation
Discretization
2-phase Action Functional
2-phase Euler Equation
2-phase Difference Euler Equation
Singularity Theory
Variation
Discretization
3-phase Action Functional
3-phase Euler Equation
3-phase Difference Euler Equation
Applied Math(VOF-method)

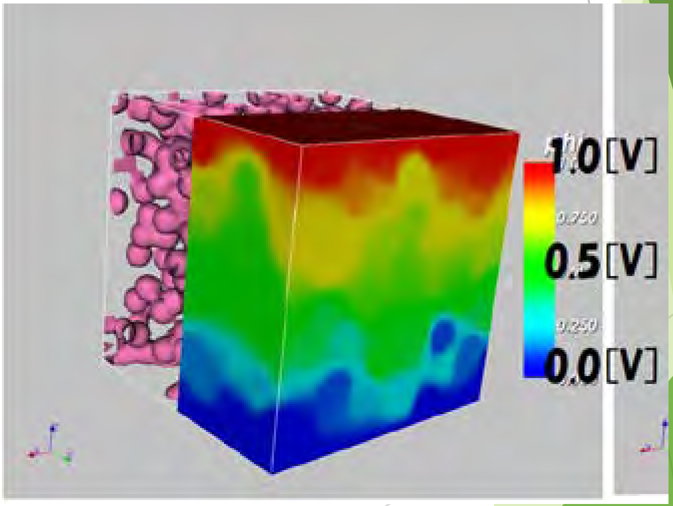
Mathematical Modeling of CFD



Adv. Math. Inves. on Conductivity in Percolation

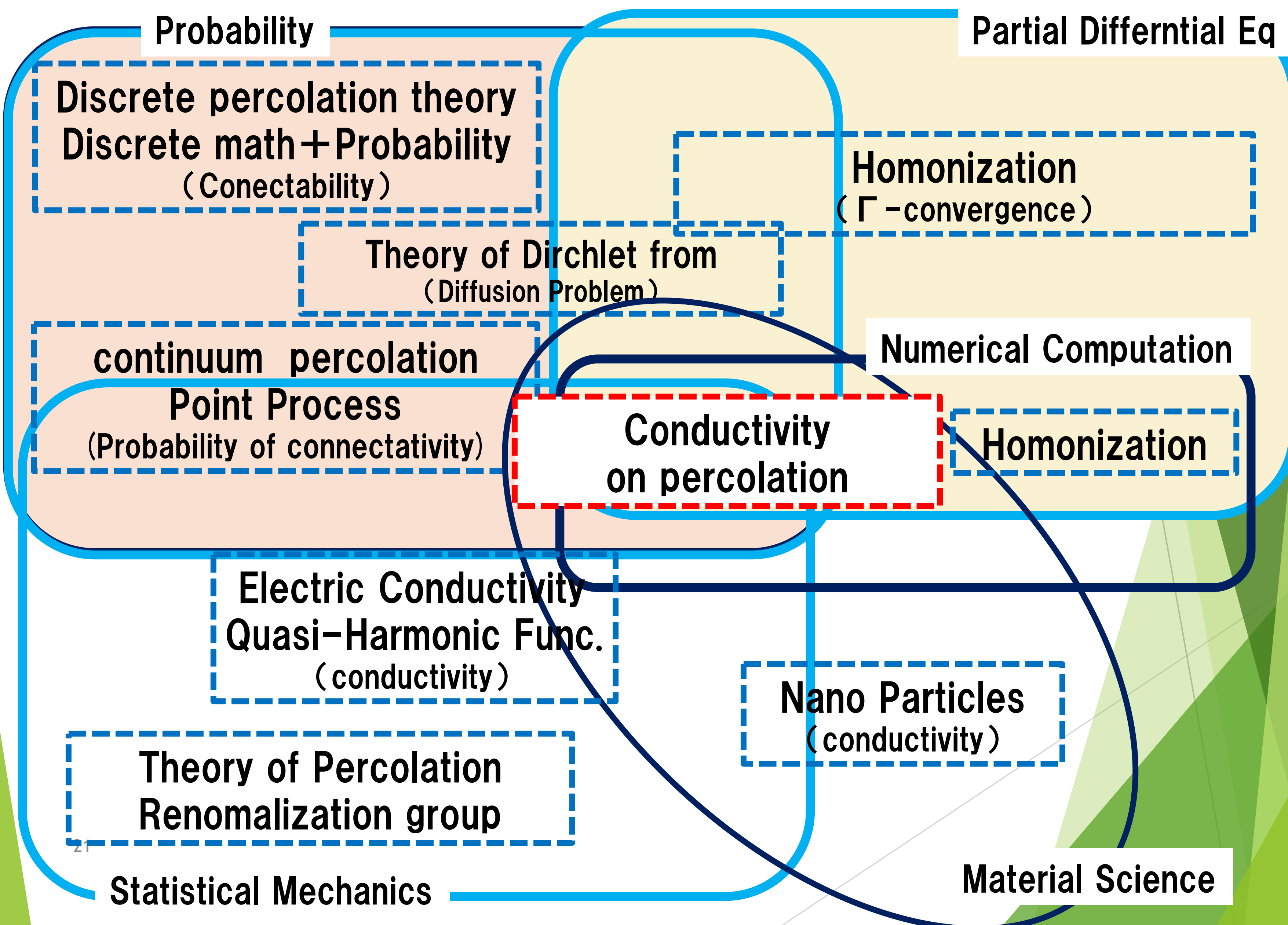
Evaluate the conductivity of random system to design the nano-materials, which is based on percolation theory, pseudo-harmonic theory, Gamma-convergence theory.
(2015 M-Shimosako-Wang)





Fine Dispersion and Property Differentiation of Nanoscale Silicate Platelets and Spheres in Epoxy Nanocomposites Chien-Chia Chu, Jiang-Jen Lin, Chang-Ru Shiu and Chang-Chin Kwan
Polymer Journal, Vol. 37 (2005) No. 4 pp.239-245

Conductivity on Percolation



Conducitivity in Continuum Percolation

Percolation

Point Process

Pseudo-conformal analysis

Fractal geometry

Partial Differential Eq.

Gamma Convergence

Measurement and Data Analysis

Material Science

Numerical Computation

Adv. Math. Inves. on Electron emitter

•To control the electron emission devices, we analyzed the emission mechanism and its properties.

1997 Okuda-M-Asai

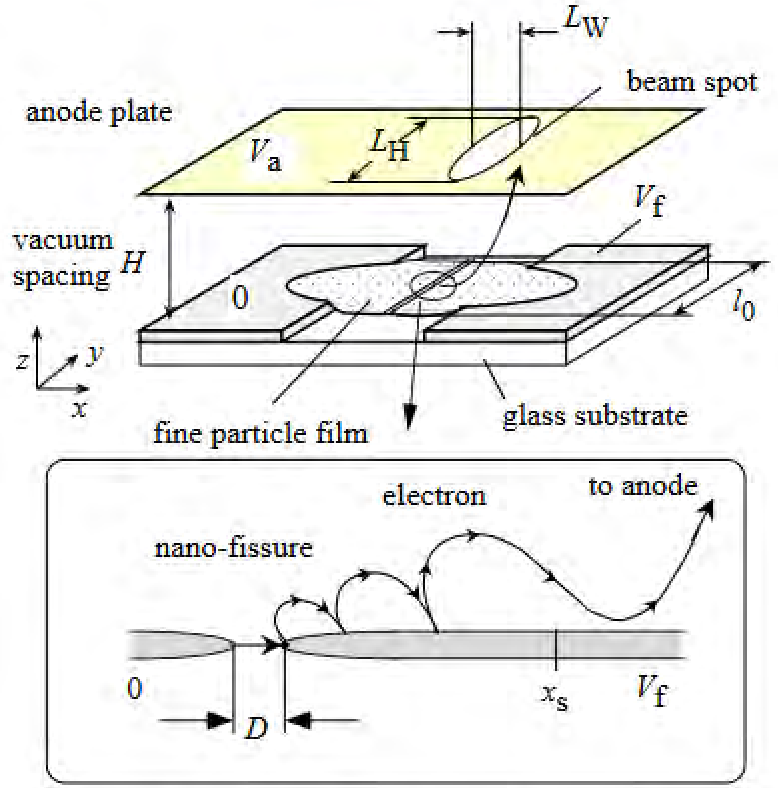


Figure 1: Schematic description of SED and multiple scattering model

Caustics
(Singularity theory)

Measurement
and
Data Analysis

Symplectic
Method

Orbit analysis

Classical
Electric Field

Electron
Devices

Fractal
geometry

Adaptive Mesh
Refinement

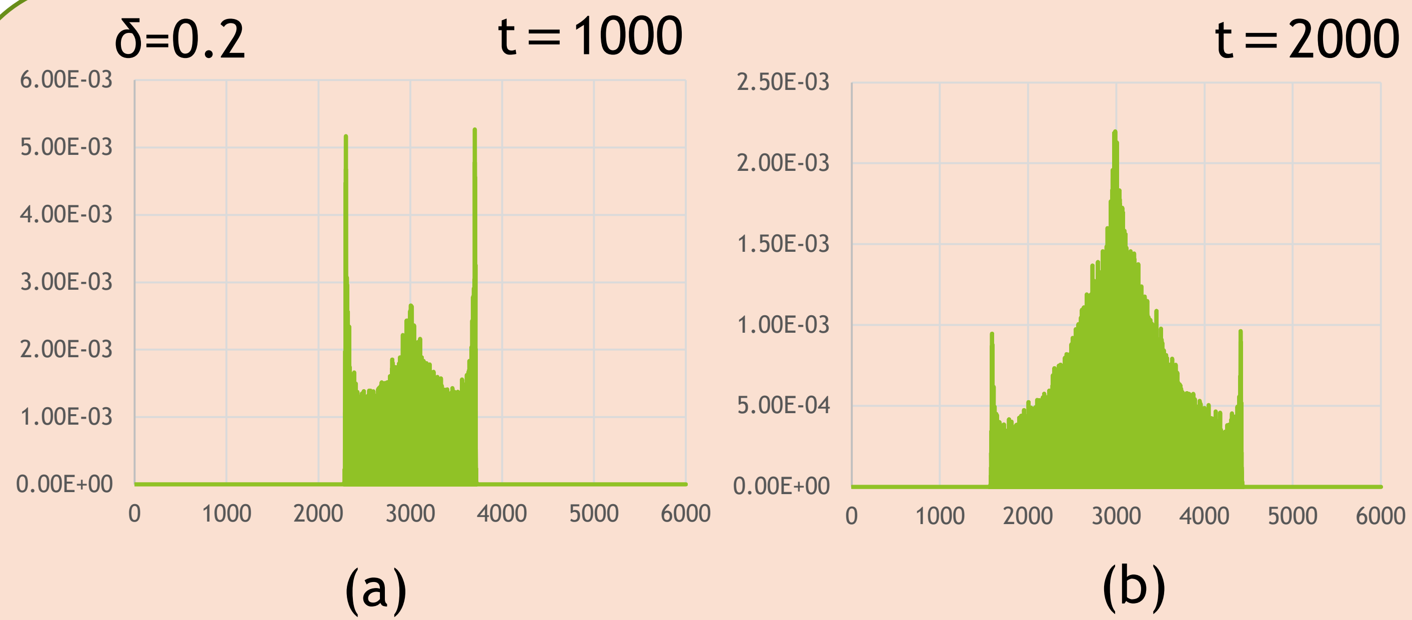
Numerical
Computation

Partial
Differential Eq.

Adv. Math. Inves. on Coloring

- To unify the diffusive color and structure color
  Quantum Walk and Point Process

2017 Ide-Konno-M-Mitsuhashi (Ann. Phys.)



Coexistence of coherent wave and diffuse wave
In quantum walk with Poisson point process impurities

Wave Optics

Point Process

Probability theory

Geometrical Optics

Coloring

Partial Differential equation

Wave in disordered system

Quanum Walk

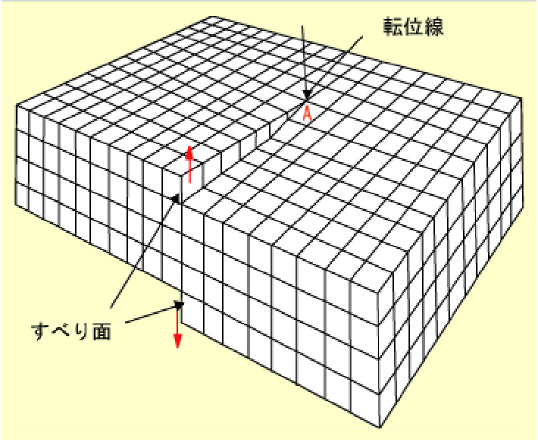
Coloring Theory

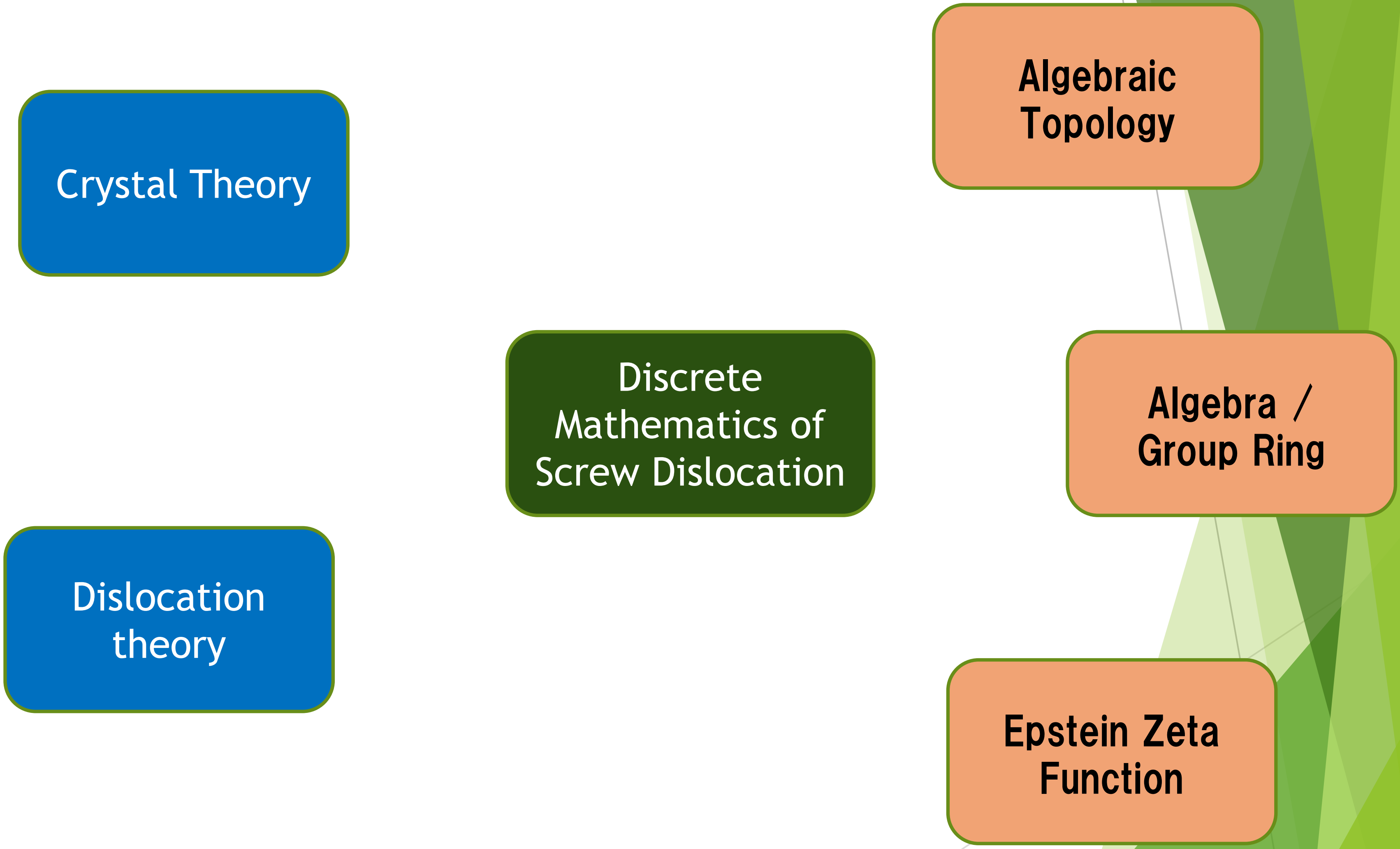
Adv. Math. Inves. on Material Structure

In order to give a direction of improvement of material design of steel, we represent the screw dislocation in material using Ableian group-ring and zeta function

$$R=C[[a_1,a_2,a_3,b]]/(a_1a_2a_3-b)$$

2016 Hamada-M-Nakagawa-Saeki-Uesaka

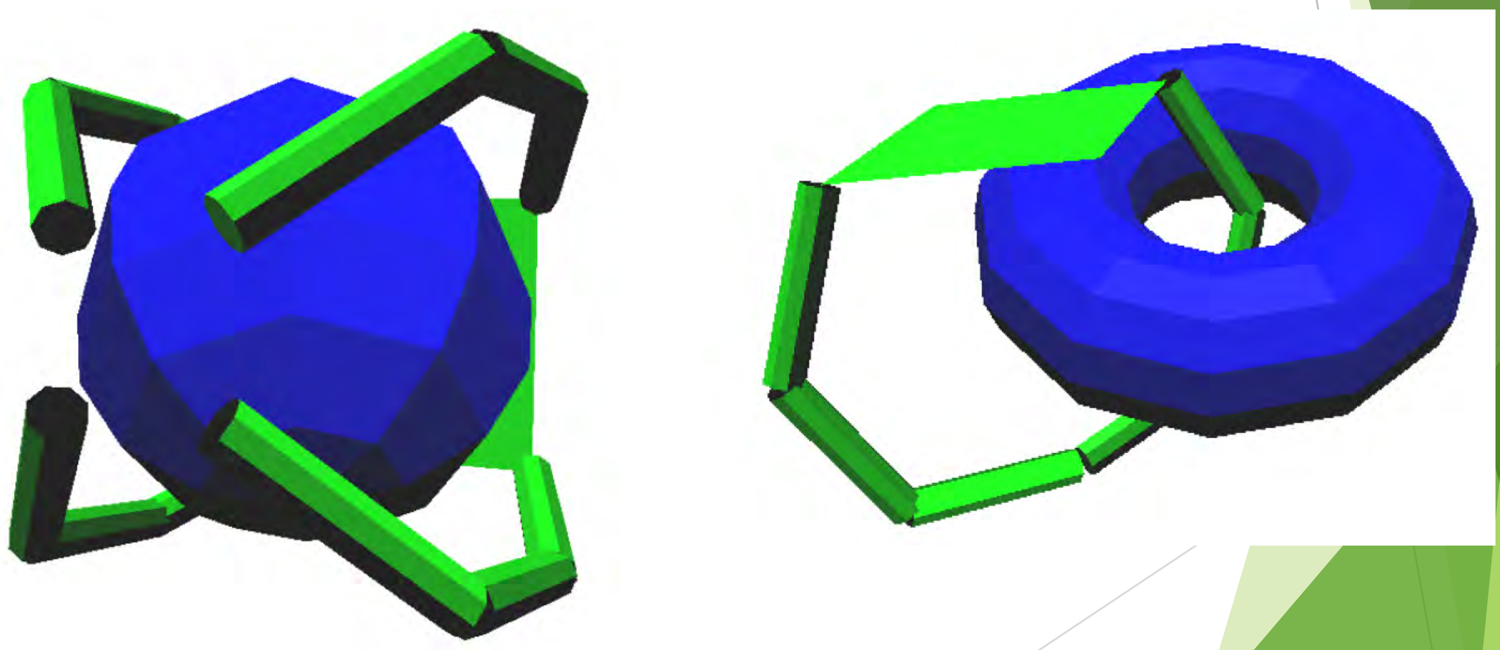



Crystal Theory
Dislocation theory
Discrete Mathematics of Screw Dislocation
Algebraic Topology
Algebra / Group Ring
Epstein Zeta Function

Adv. Math. Inves. on Robtics

In order to give a new constrant method in robotics,
We need a mathemaitical field which is mixed of SE(n)
eucliedean moves and homotopy theory.

2017 Hamada-Makita-M (JGSP)



Homotopy theory

Robtics

Caging theory

Euclidean moves

Control theory

Geometry of Path space

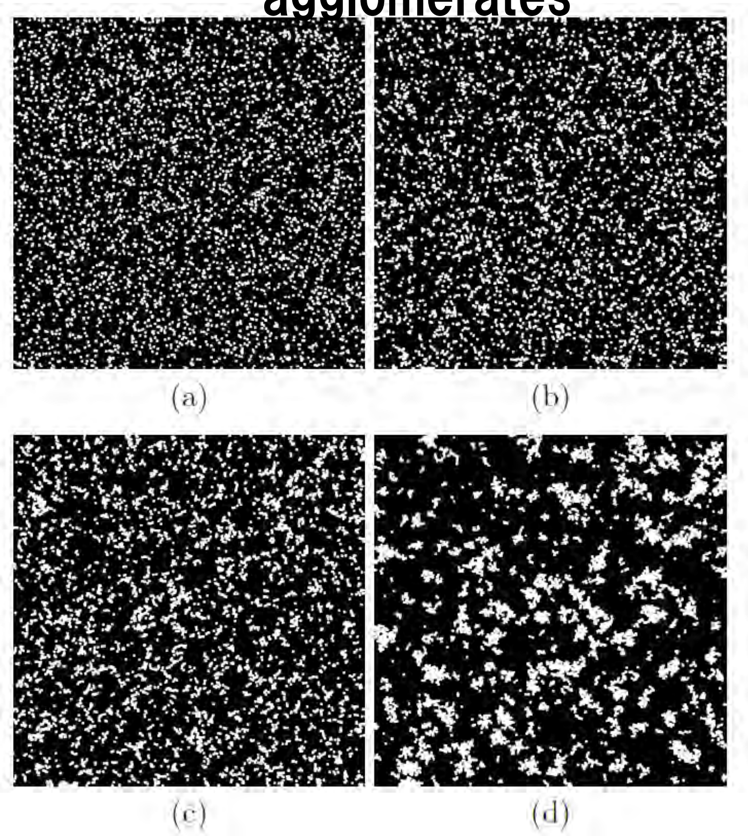
Kakeya Problem, Coxeter problem

Adv. Math. Inves. on Morphology

In order to give a realistic value of morphology of agglomerating particles in SEM data.

M-Shimosako

agglomerates



Morphology

Image Analysis

Morphology of

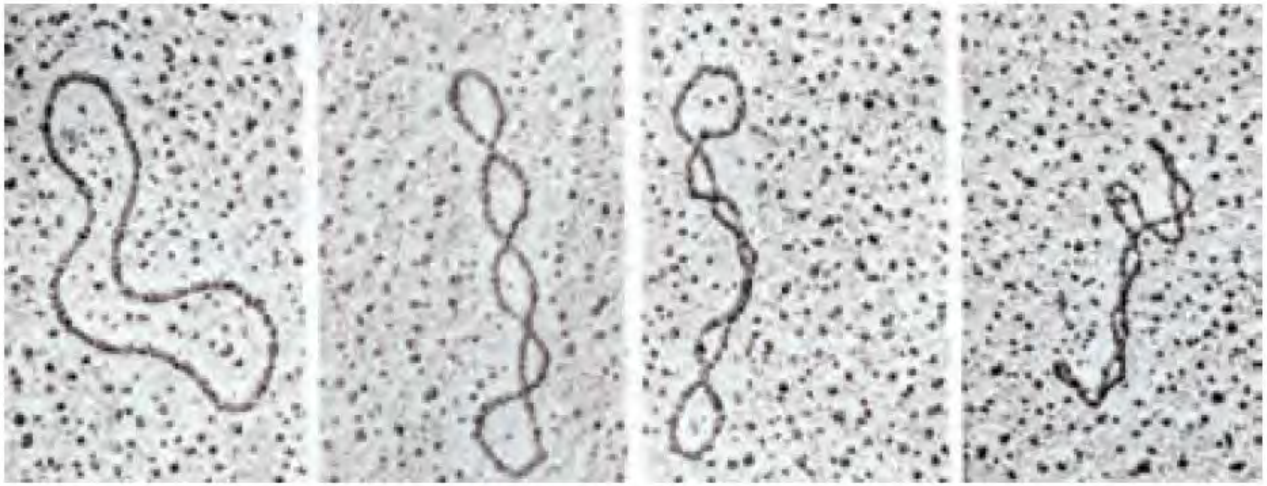
Point Process

Persistent Homology

Adv. Math. Inves. on shape of DNA

In order to represent the shape of DNA, we consider the statistical mechanics of DNA and investigate the Ableian function theory

M (1997), M-Onishi (2003), M-Previato(2016)



http://www.udel.edu/chem/bahnson/chem645/websites/Sapra/Supercoiling.html

Invesigation of Immersion of S^1 into complex plane

Integrable System

Theory of Elastic Body

Abelian Function Theory

Statistical Mechanics of elastica

Algebraic Curves

Path Integral Formulation of Quantum Field Theory

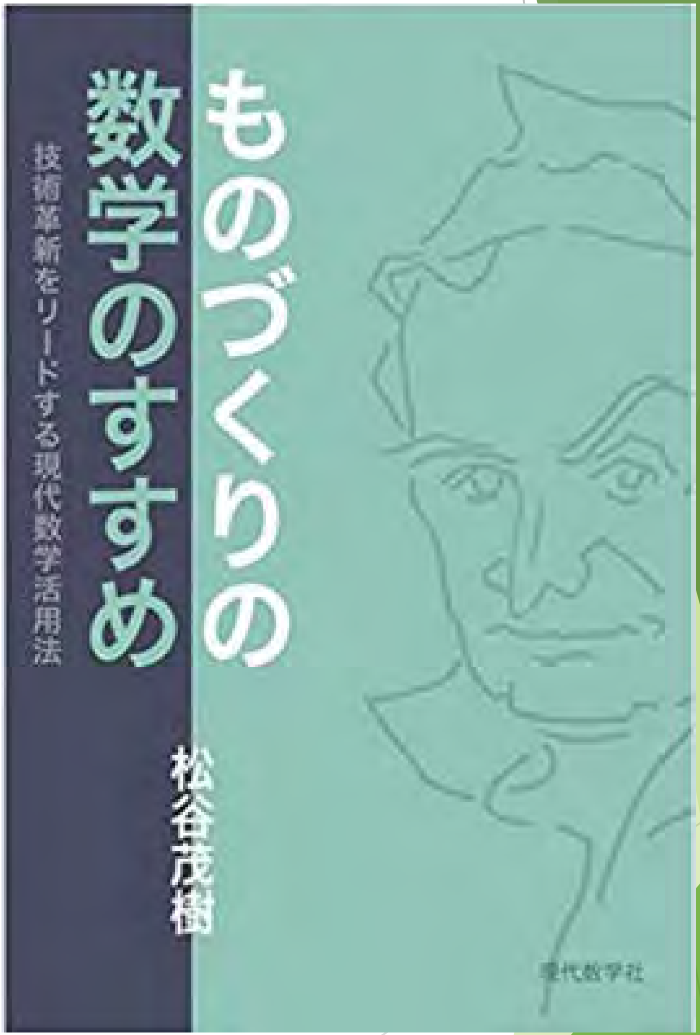
Geometry of Loop Space

infinite dimensional Lie algebra

Leonardo da Vinci said "*Mechanics (Technology) is the paradise of the mathematical sciences because by means of it one comes to the fruits of mathematics.*"

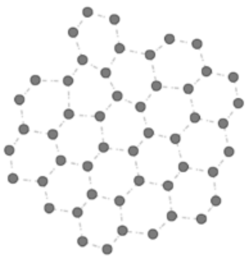
In XXI century, we have so many cases that mathematics has the strong power for industrial studies.
In each company, there are such examples but they are not disclosed due to commercial secret.
In Japan, since there are few human exchanges among academia and companies, mathematics for industry might not be so well-established.

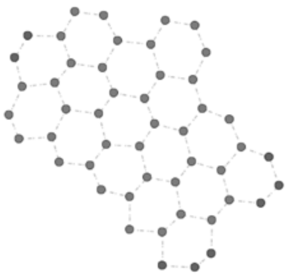
線型代数周遊ー応用をめざして

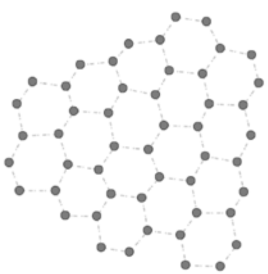


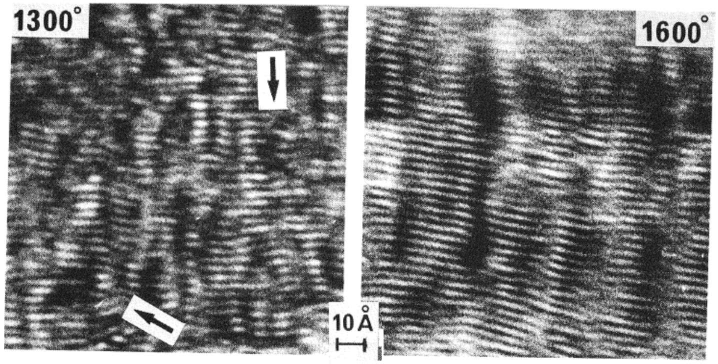
Adv. Math. Inves. on Conductivity of Disordered Carbon

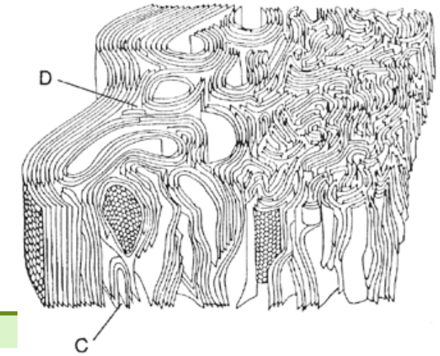
graph ξ function, Random Matrix theory Percolation theory
Phys.Lett. A (2017M-Sato)

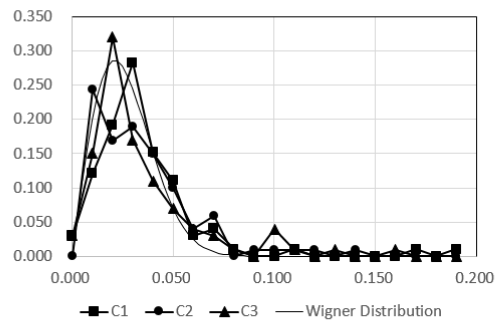


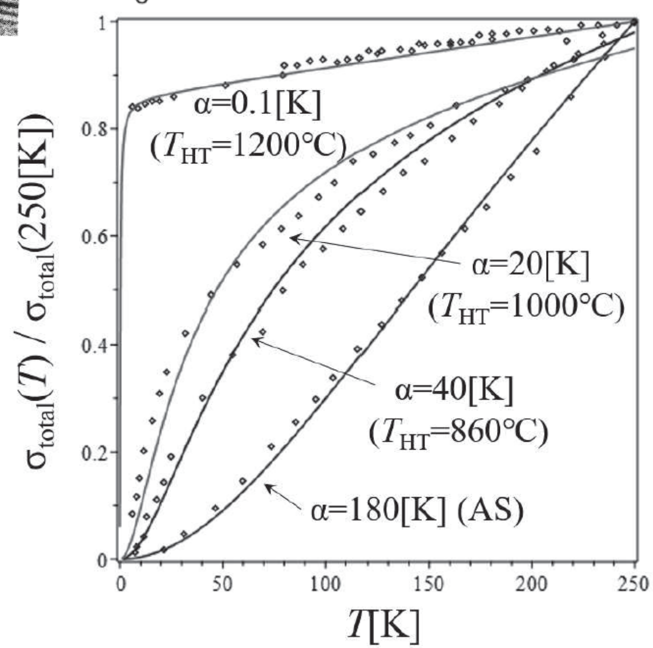












Carbon

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2. S. Matsutani and Akira Suzuki, Apparent metal-insulator transition in disordered carbon, Phys. Rev. B, 62 (21) (2000) 13812-13815.
3. S. Matsutani and Iwao Sato, *A novel conductivity mechanism of highly disordered carbon systems based on an investigation of graph zeta function* Phys. Lett. A **381** 36, (2017), 3015-3140.

Percolation

1. S. Matsutani, Y. Shimosako, and Y. Wang, *Numerical computations of conductivity in continuum percolation for overlapping ellipsoids*, Int. J. Mod. Phys. C **21** (6) (2010) 709-729.
2. S. Matsutani, Y. Shimosako and Y.Wang, *Fractal Structure of Equipotential Curves on a Continuum Percolation Model* Physica A **391** (23) (2012) 5802-5809,
3. S. Matsutani, Y. Shimosako and Y. Wang, *Numerical Computations of Conductivities overAgglomerated Continuum Percolation Models*, Applied Math. Modeling **37** (2013) 4007-4022.
4. S. Matsutani and Y. Shimosako, *On homogenized conductivity and fractal structure in a high contrast continuum percolation model* Applied Mathematical Modelling **39** (2015) 7227-7243,

Today' s topic is also one of examples of Advanced Mathematical Investigation.

In order to show the importance of the advanced mathematical investigation,
I showed it as one of examples, with Prof. Iwao Sato. (PLA 2017), which is a revision of the study with Prof. A. Suzuki (PLA 1997, PRB 2000).
This was related to the analysis of the electron emitter.

Properties of Carbon

Quantum Chemistry
（tight binding近似）

Quantum Mechanics of Disordered system

Electric Conductivity in Disordered system

Electric Conductivity

Conductivity of Disordered Carbon

Conductivity of Continuum Percolation

Random Matrix Theory

Theory of graph zeta function

Graph theory

Lattice Green Function

Advanced Mathematical Investigation for conductivity of highly disordered carbon systems; percolation and graph zeta function

1. Activation carbon fiber
2. Conductivity of ACFs
   Kuriyama's Investigation
3. Conductivity of percolation
4. Graph Theory
5. New proposals on the conductivity
6. Summary

Advanced Mathematical Investigation for conductivity of highly disordered carbon systems; percolation and graph zeta function

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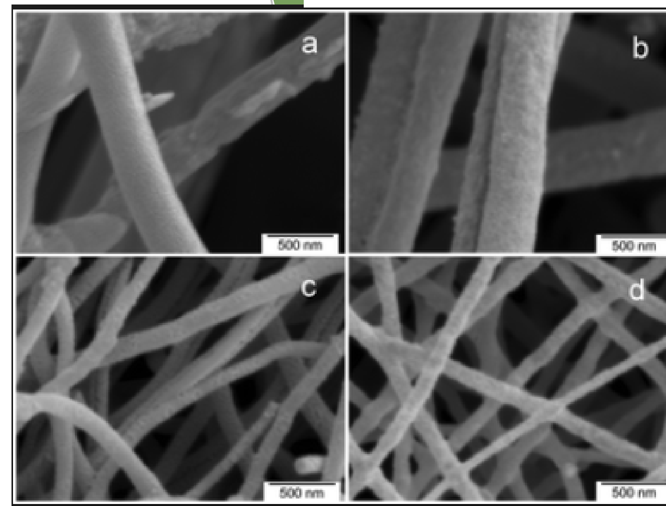
3. Conductivity of percolation
4. Graph Theory
5. New proposals on the conductivity
6. Summary

S. Hua and Y.-L. Hsieh
J. Mat. Chem. A 37, 2013

Activated Carbon fiber is the carbon fiber with radius 2-15[um]

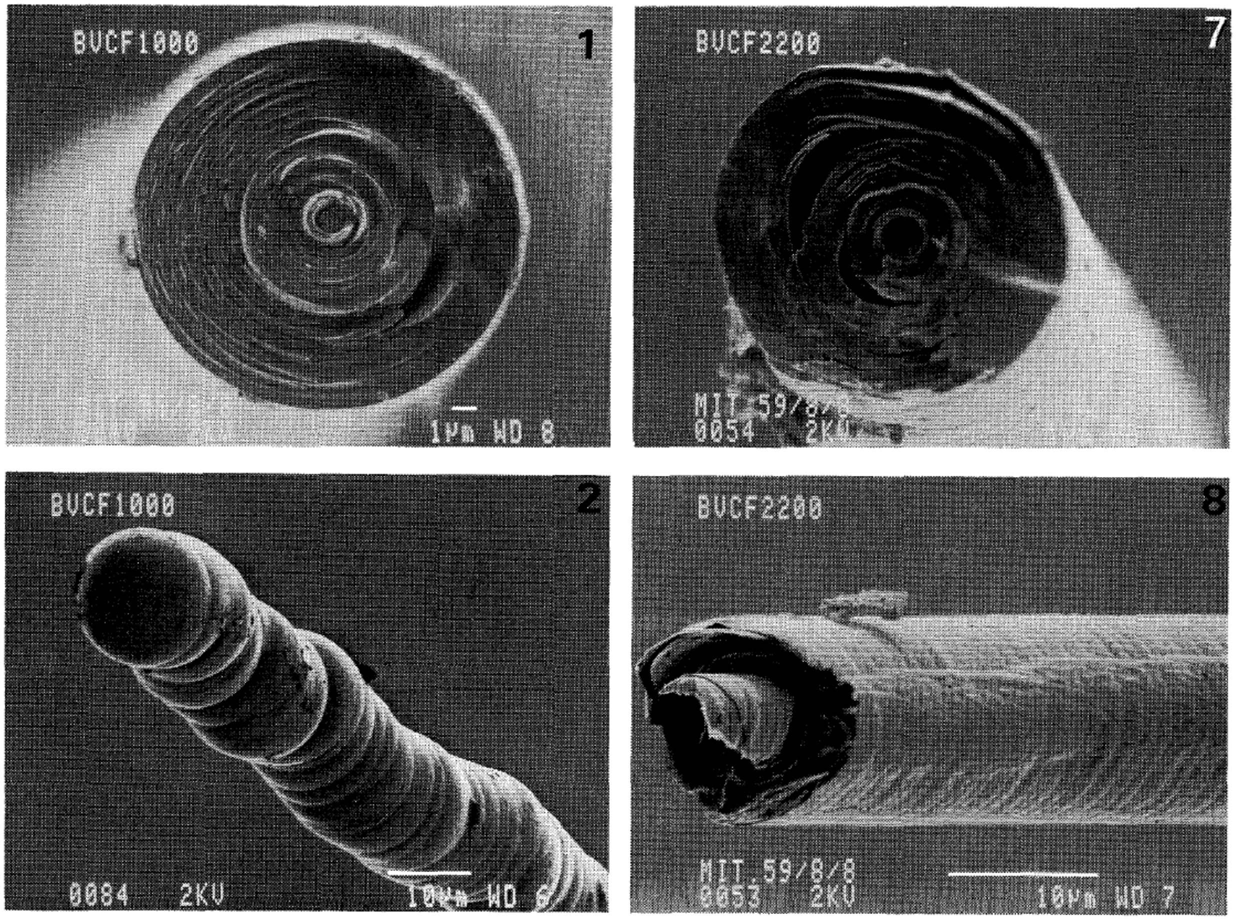


Carbonized at 770K

Cellulose/ PAN (Polyacrylonitrile) → **N2 flow** → Carbon Fiber → **Activation (with gas) e.g. Ar gas** → Activated Carbon Fiber

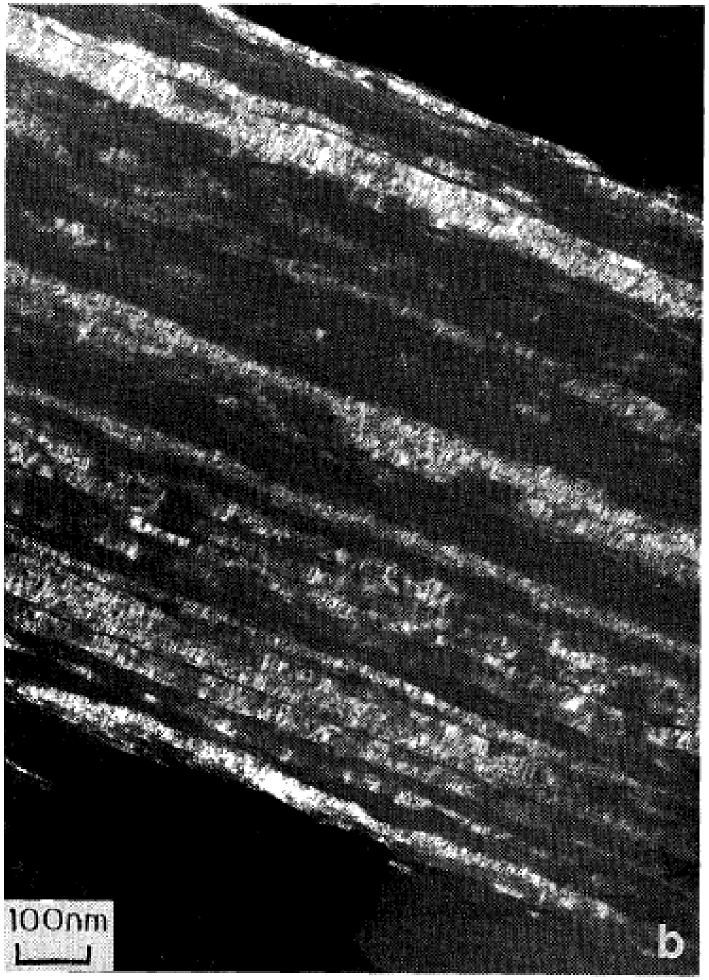
http://www.utsi.edu/index.php/utsi-activated-carbon-fibers-acfs/

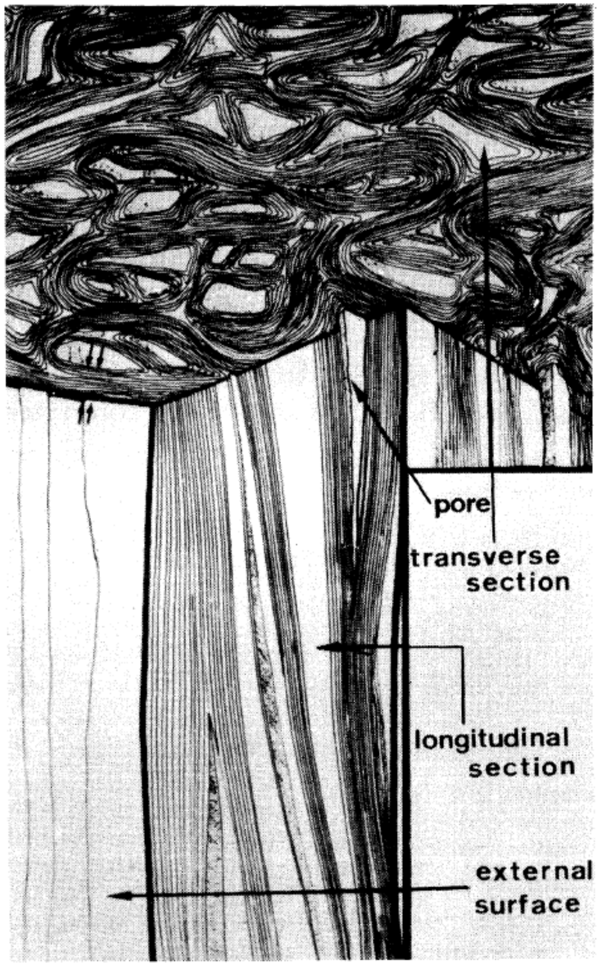
Several 10 micron meter structure



Several 10 [um]

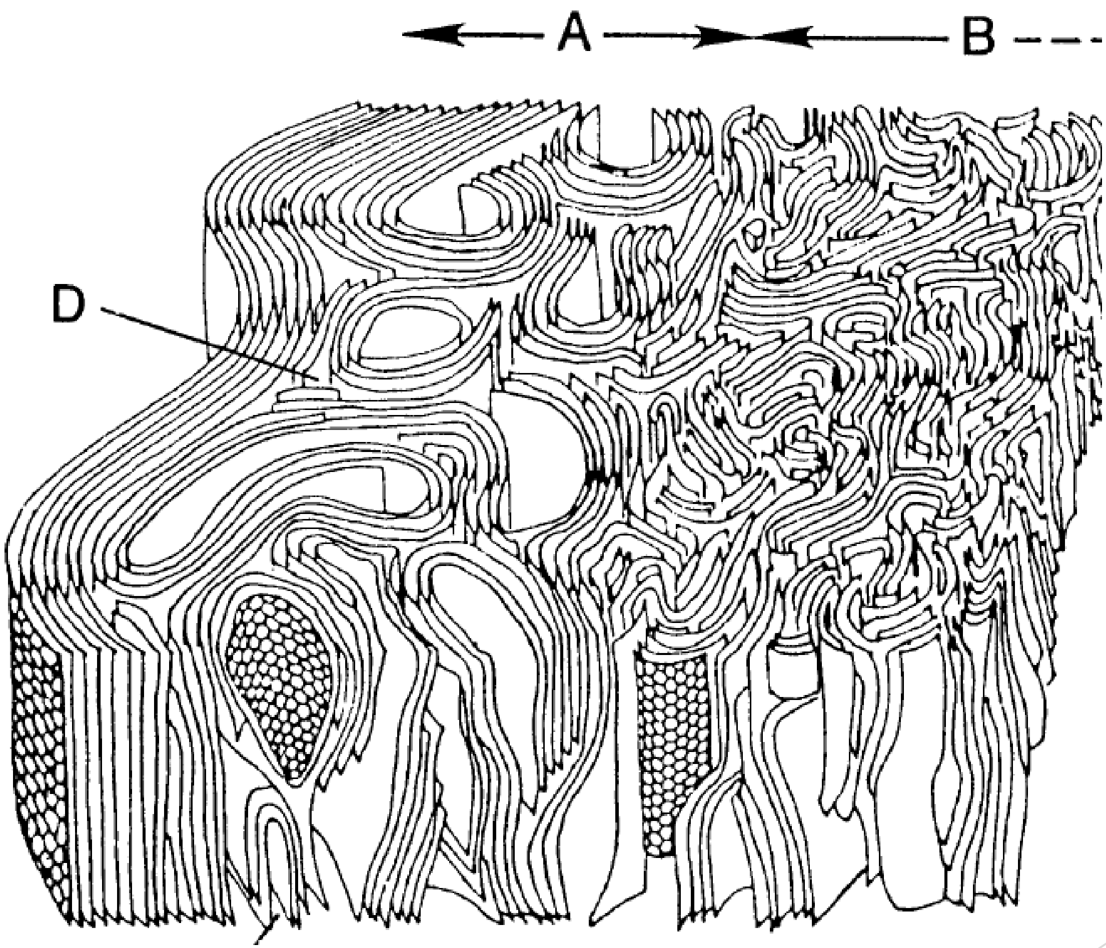
Sub-micron meter structure



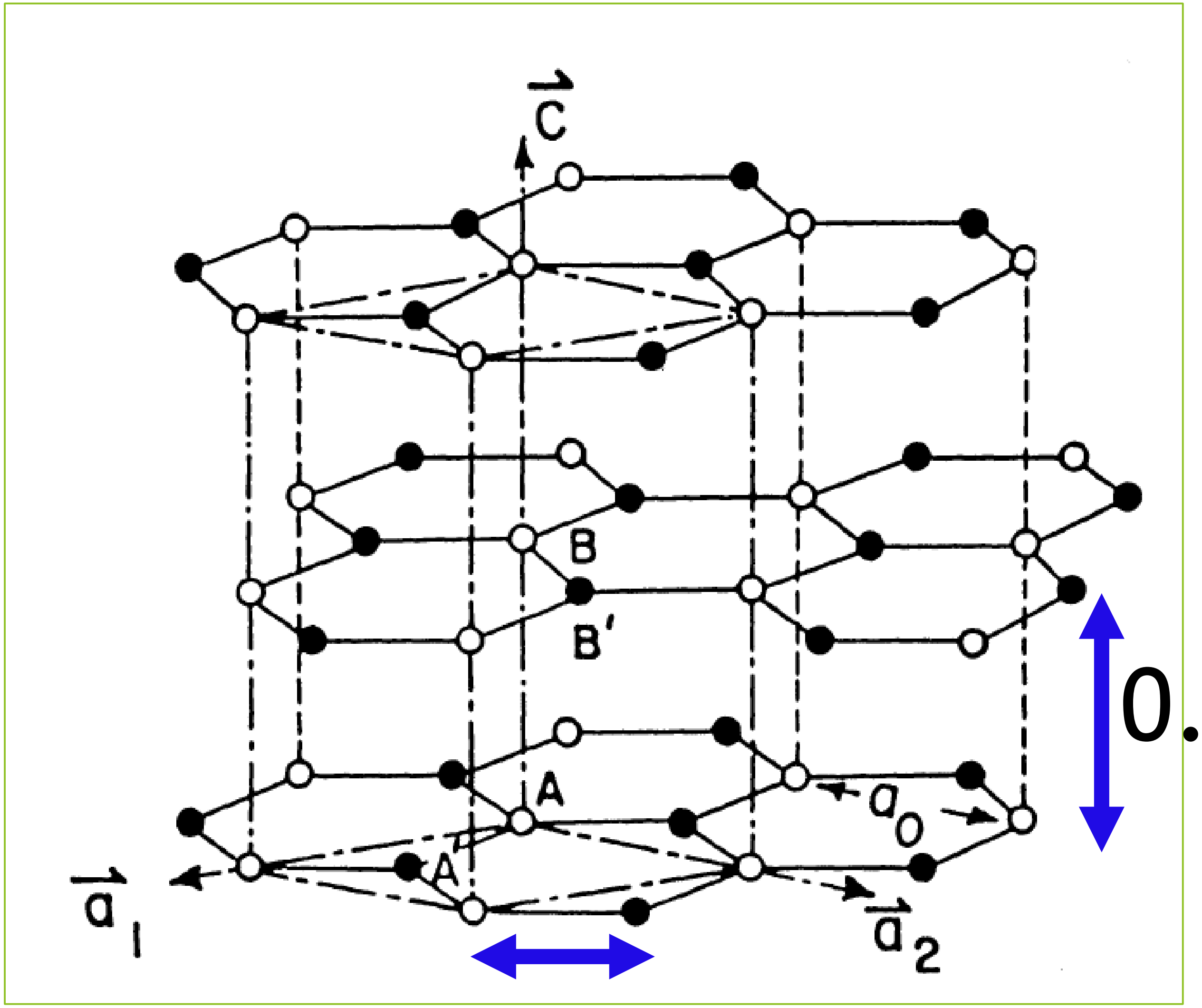


It is stratified!

Nano meter structure



Complicated stratified structure appears

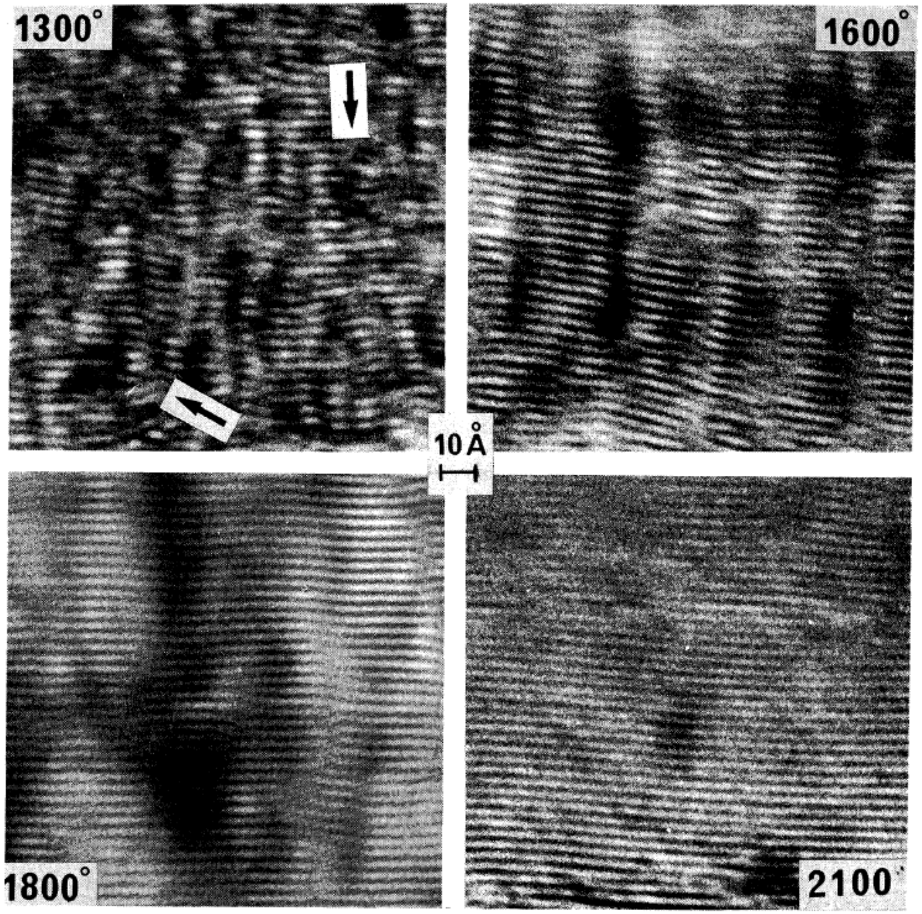


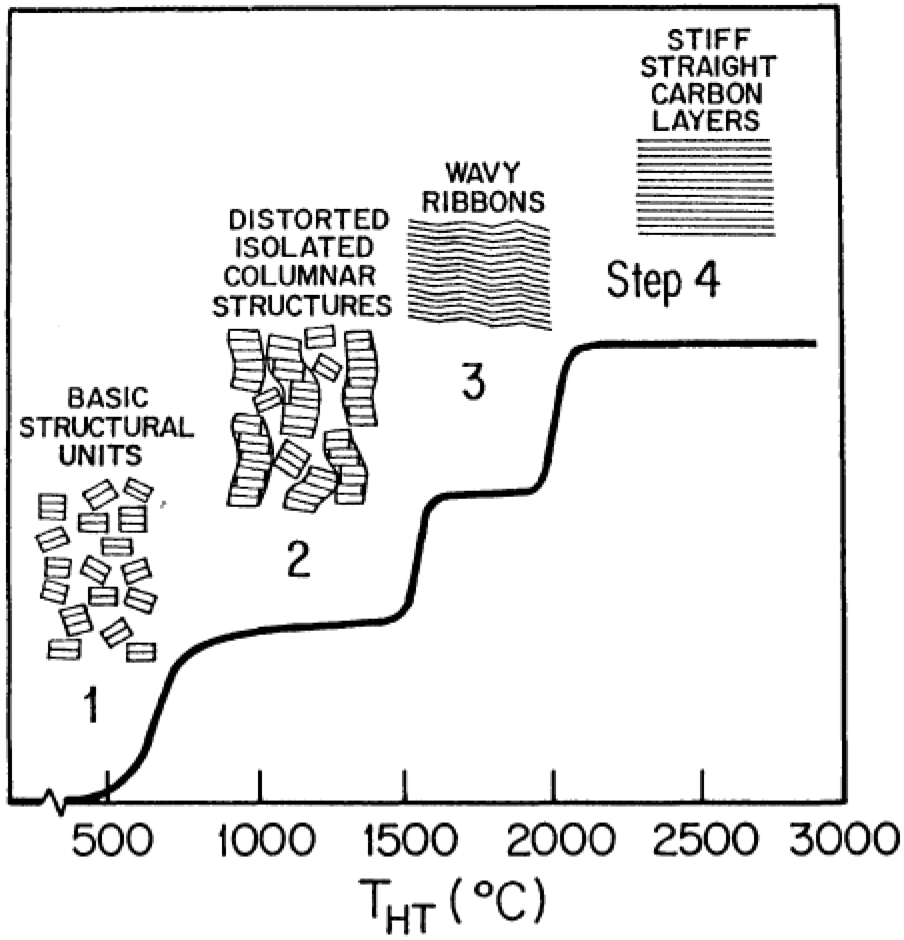
0.3354[nm]

0.1421[nm]

Heat treatment (e.g. with At gas 1 hour) makes the micro structure change

The higher temperature is, the size of pieces is larger!





Advanced Mathematical Investigation for conductivity of highly disordered carbon systems; percolation and graph zeta function

Kuriyama showed his experimantal results of conductivity of ACFs and proposed a new mechanism of the conductivity.

Phys.Rev.B 1993

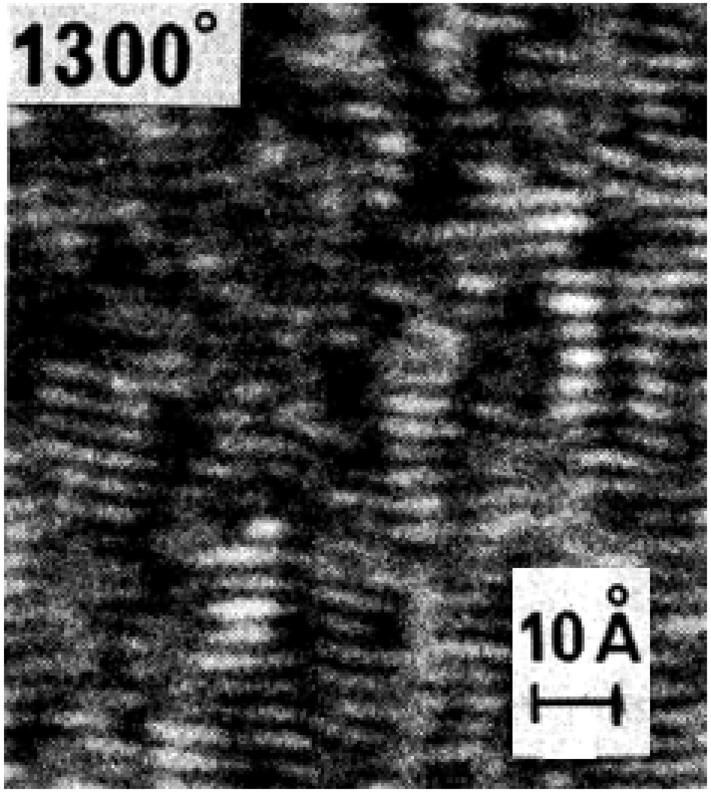
PHYSICAL REVIEW B VOLUME 47, NUMBER 19 15 MAY 1993-I

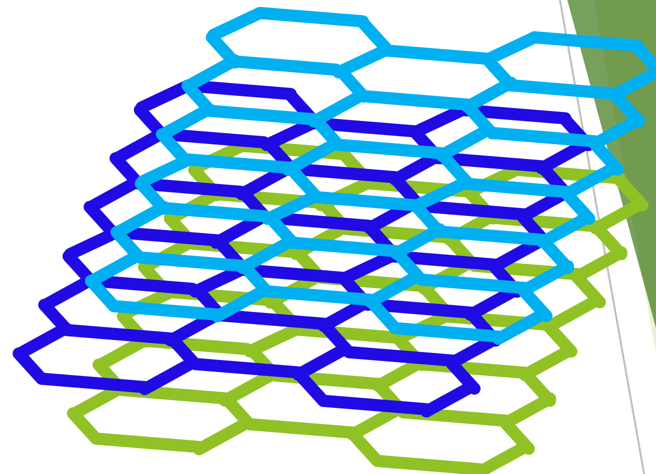
Hopping conductivity with distributed energy-barrier heights

K. Kuriyama
Research Center, Sumitomo Metal Industries, Fuso 1-8, Amagasaki 660, Japan

(Received 24 September 1992)

An alternative expression for the temperature dependence of hopping conductivity is proposed. A conduction model is proposed based on a collection of many independent Arrhenius-type processes. The density in the material considered has a Λ-shaped distribution as a function of activation energy, for example Gaussian, isosceles, and scalene distributions. The validity of the model has been checked with the electrical-conductivity data of disordered carbon fibers which show a metal-insulator transition. The result is that the conductivity data between 4 and 250 K fit well to the form $T^2[1-\exp(-E/kT)]^2$, where T is temperature and E activation energy, and is related to the degree of disorder in the system. This form is the simplest form derived from the isosceles distribution; however, a better fit is obtained from the scalene distribution with more complex form. By using the proposed conduction model, the activation energy E is found to decrease systematically as the insulator-metal transition is approached by heat treatment. Also this model can yield the widely observed fractional temperature dependences of hopping conduction: $x=1, \frac{1}{2}, \frac{1}{3}$, and $\frac{1}{4}$ in the conventional form $\exp-(T_0/T)^x$.





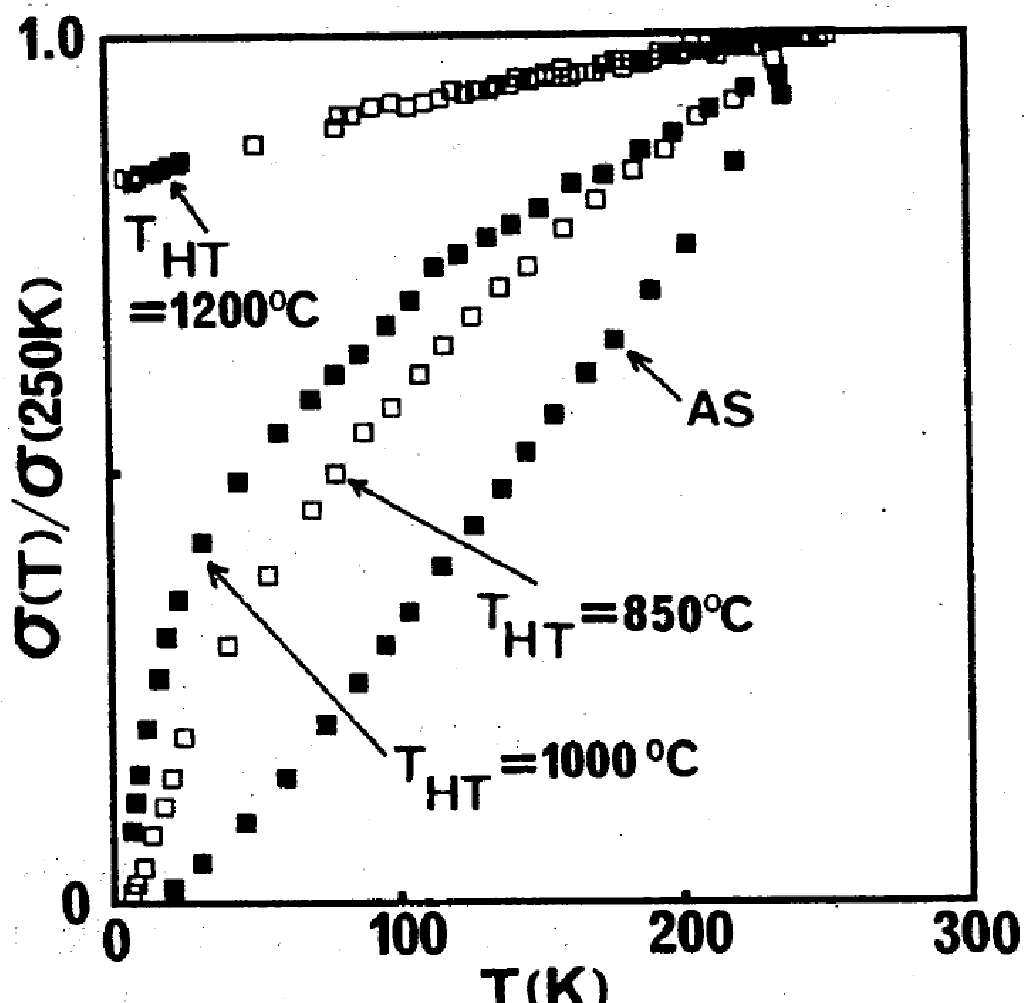
1.0 x 10^3 atoms

- ACF is so prous, and consiss of randamaized granue 3 sheat（due to Laman 1.1nm peak）with La=3.0 [nm]
- Mesurament: Four Proved Method.
- Conductivity ～10 S/cm（～10^{-4}metal）

siemens

Kuriyama 's experimantal results of conductivity of ACFs PRB 1993

Temperature Dependence of the electric conductivity parameterized by heat treatments



Mott Plot: $\sigma = \sigma_0 / T^{\gamma}$

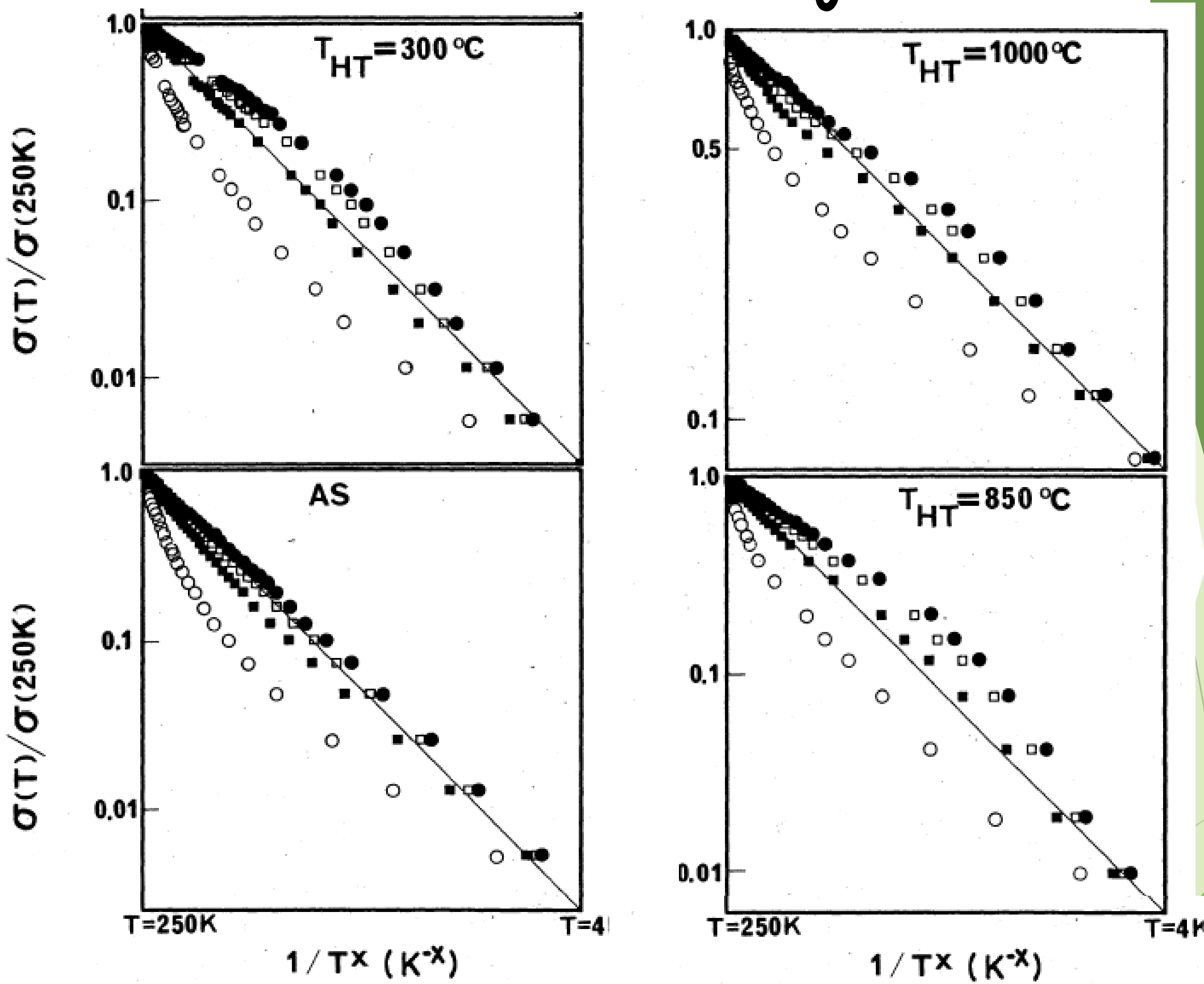
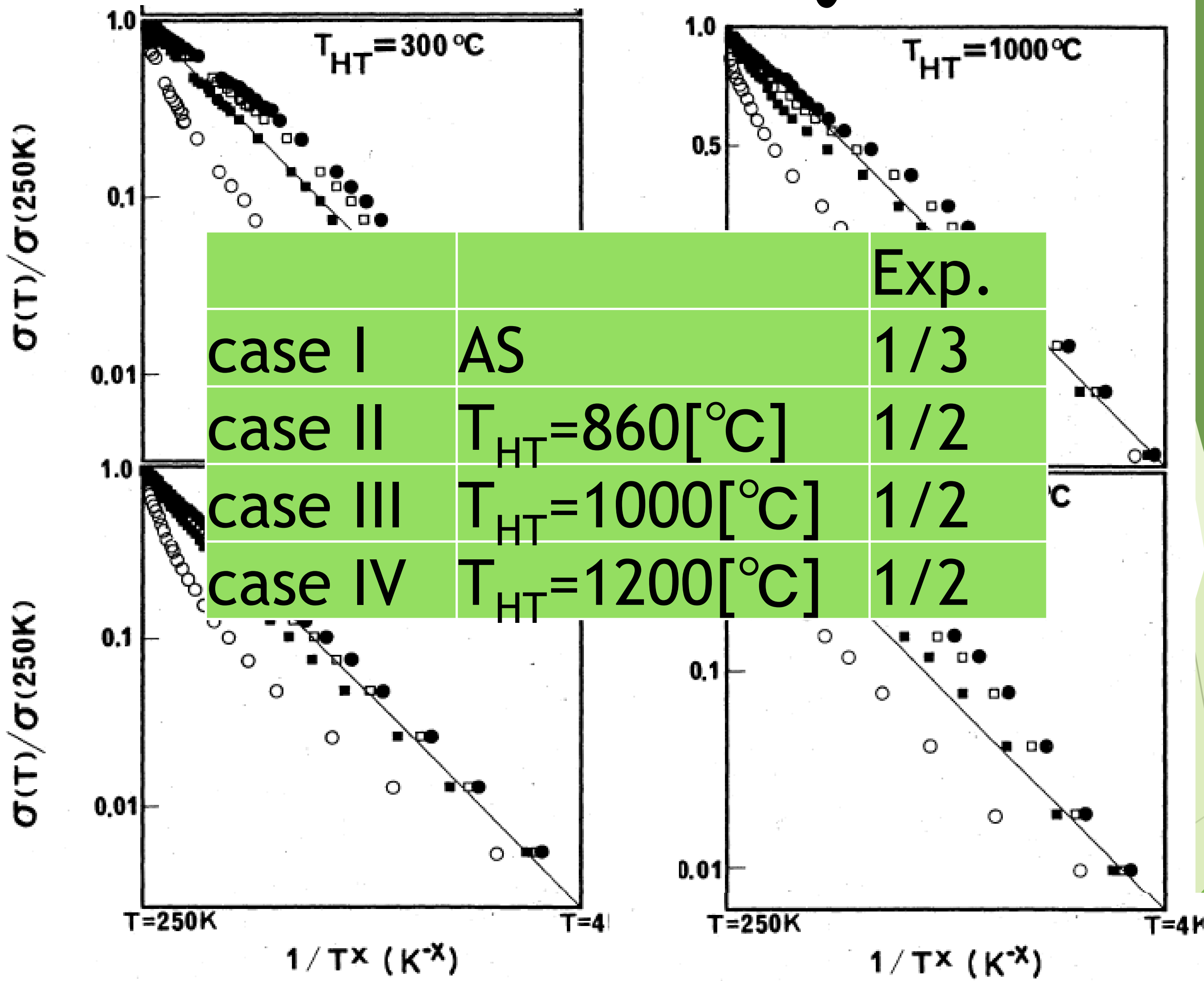


FIG. 2. The $\log_{10}$ plot of $\sigma(T)/\sigma(250\ \mathrm{K})$ vs $1/T^x$ with $x = \frac{1}{1}$ (○), $\frac{1}{2}$ (■), $\frac{1}{3}$ (□), and $\frac{1}{4}$ (●) from the left to the right.

Mott Plot: $\sigma = \sigma_0 / T^\gamma$



		Exp.
case I	AS	1/3
case II	T_{HT}=860[℃]	1/2
case III	T_{HT}=1000[℃]	1/2
case IV	T_{HT}=1200[℃]	1/2

FIG. 2. The $\log_{10}$ plot of $\sigma(T)/\sigma(250\ \mathrm{K})$ vs $1/T^x$ with $x=\frac{1}{1}$ (○), $\frac{1}{2}$ (■), $\frac{1}{3}$ (□), and $\frac{1}{4}$ (●) from the left to the right.

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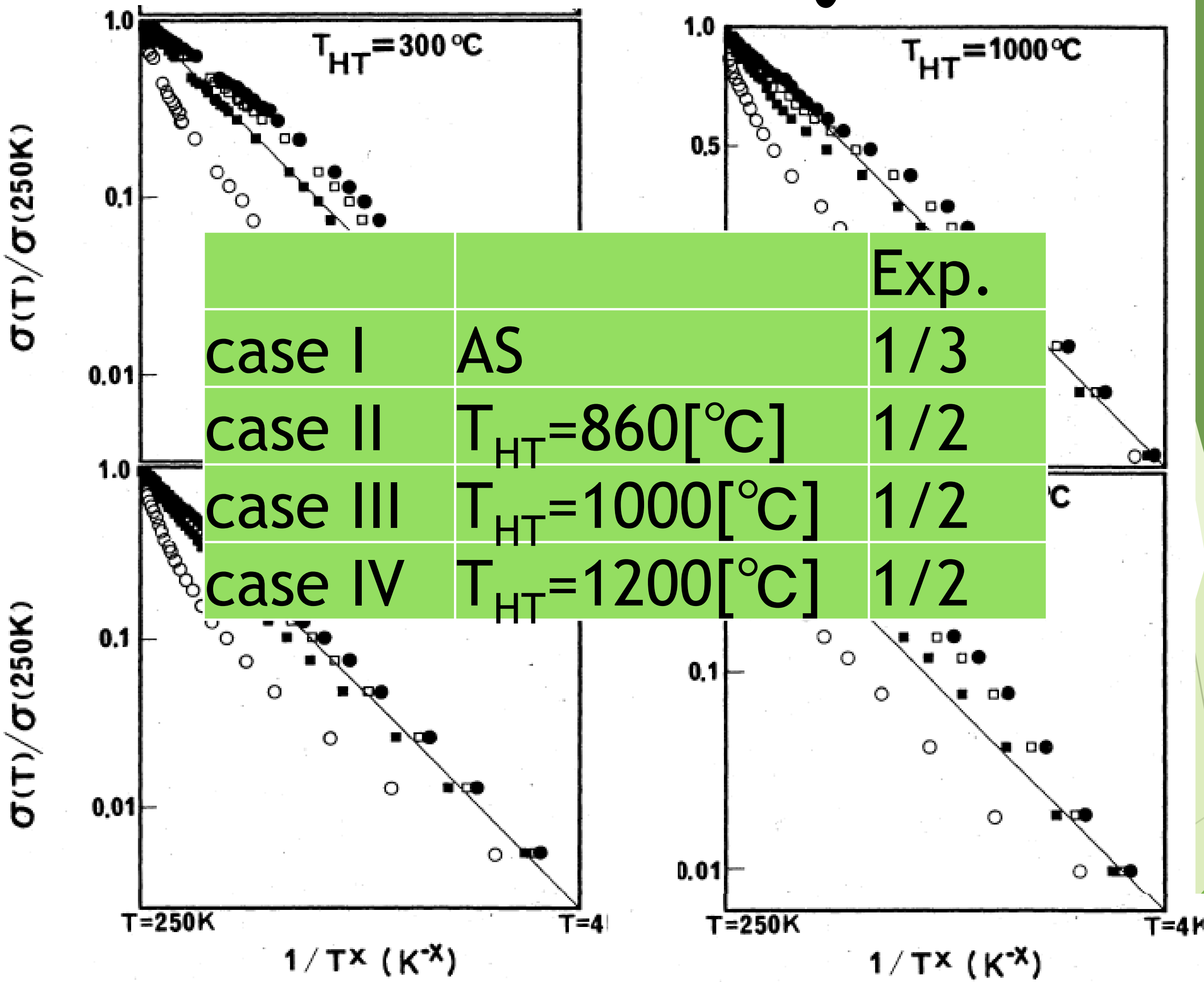

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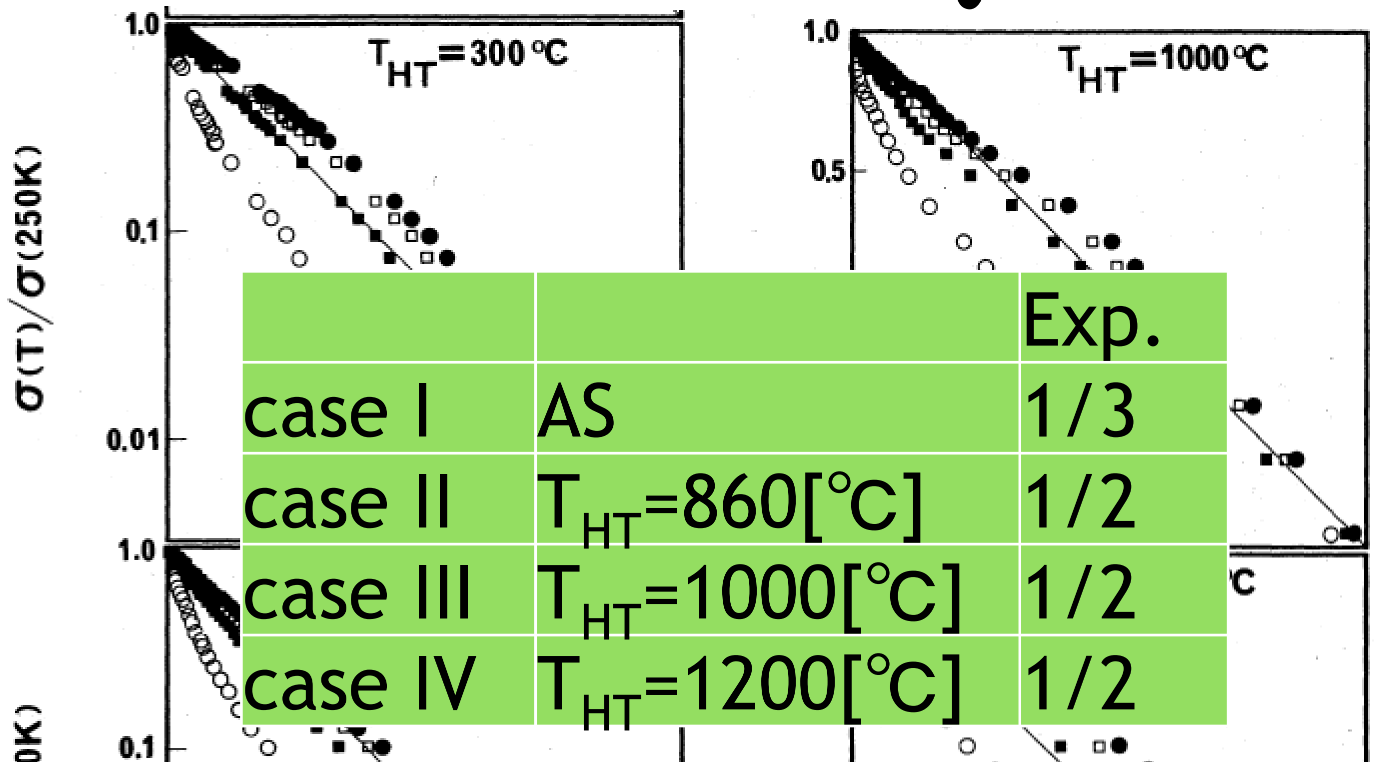
FIG. 2. The $\log_{10}$ plot of $\sigma(T)/\sigma(250\ K)$ vs $1/T^x$ with $x=\frac{1}{1}$ (○), $\frac{1}{2}$ (■), $\frac{1}{3}$ (□), and $\frac{1}{4}$ (●) from the left to the right.

Mott Plot: $\sigma = \sigma_0 / T^{\gamma}$



		Exp.
case I	AS	1/3
case II	T_{HT}=860[°C]	1/2
case III	T_{HT}=1000[°C]	1/2
case IV	T_{HT}=1200[°C]	1/2

Mott variable range hopping theory shows that

$$\sigma = \sigma_0 / T^{\gamma}$$

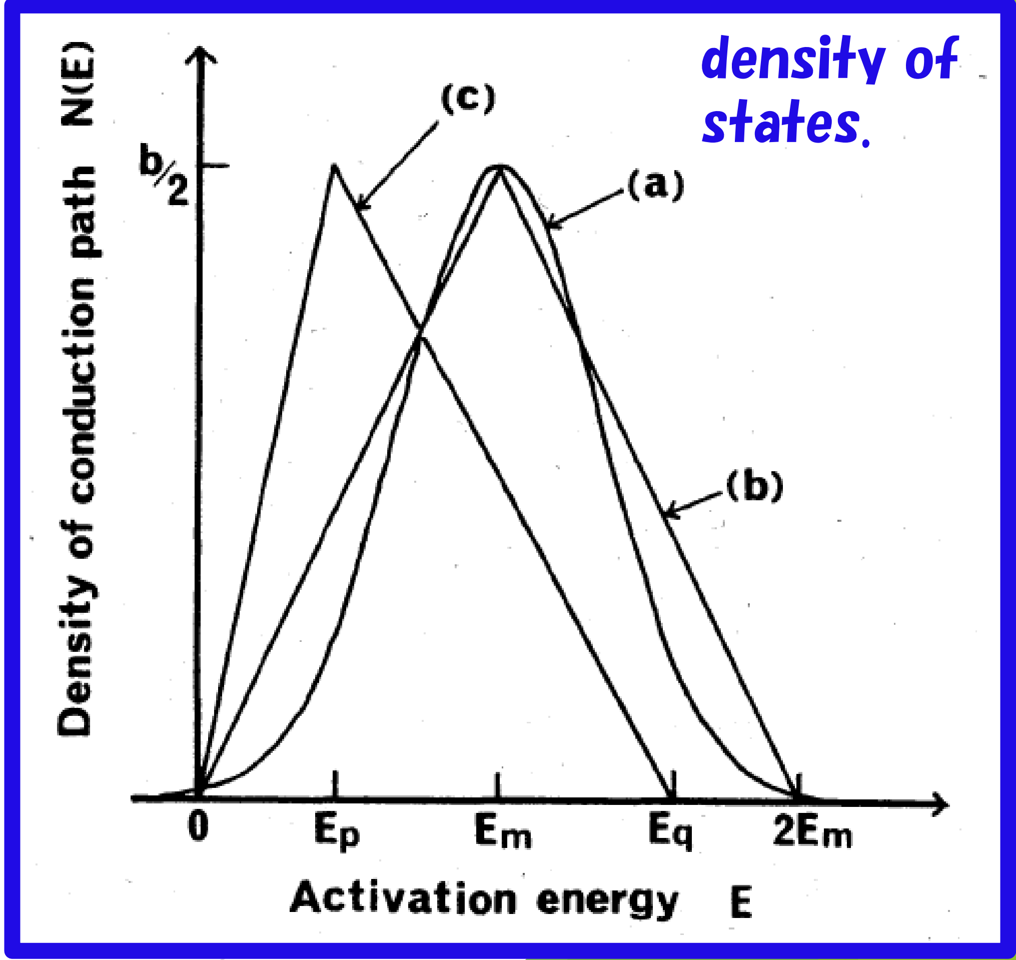
and $(\gamma - 1)$ is the dimension of subspace which electrons move.

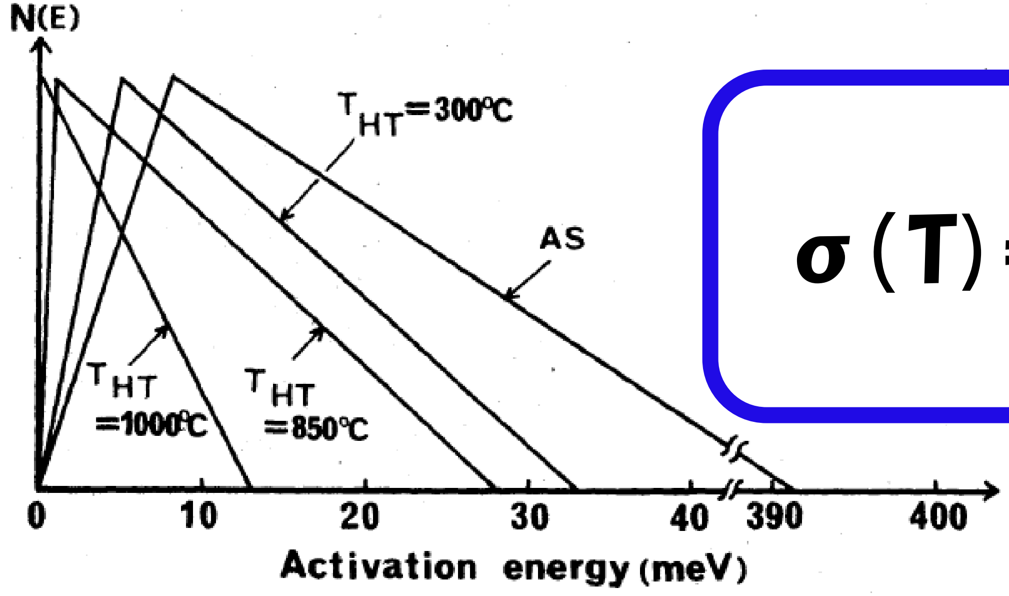
But the system must be two or three dimensional!

The best fit is known by agreement with the straight solid line.

Kuriyama found the phenomenological results on the electric conductivy!
The conductivity σ of ACFs is given by an activational phenomenon with density of states.

$$\sigma(T) = \sigma_0 \int N(E) e^{-E/T} dE$$

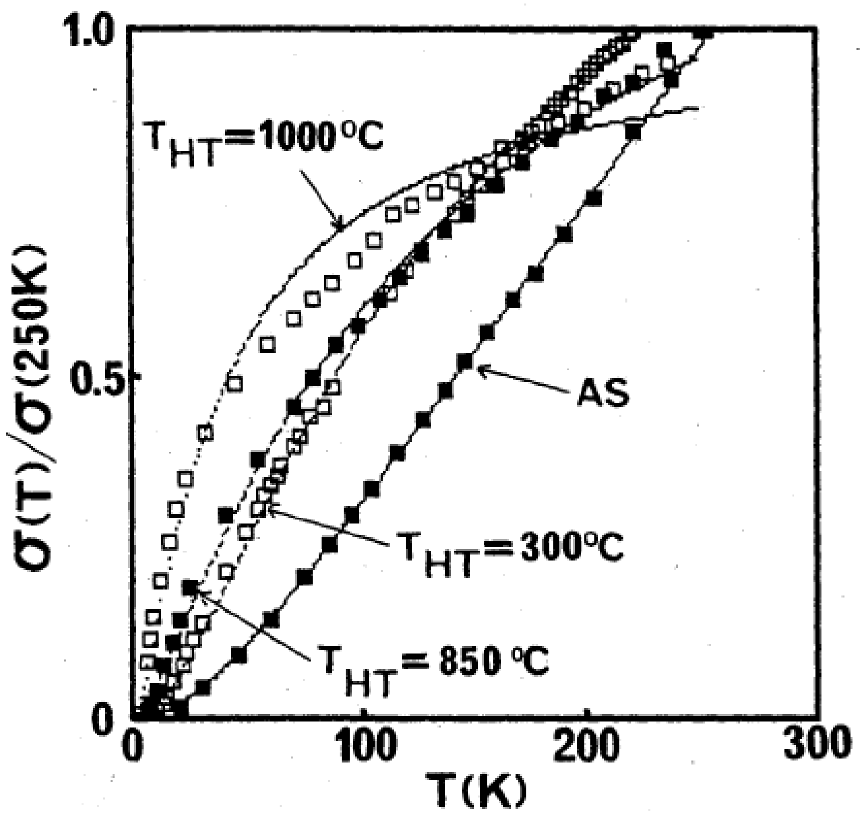




$$\sigma(T) = \sigma_0 \int N(E) e^{-E/T} dE$$

Kuriyama 's proposal recovers his experimental results

Phys.Rev.B 1993



Purpose of this Study is

1) To give the microscopic origin of the Kuriyama's mechanism, and
2) To improve his results

Advanced Mathematical Investigation for conductivity of highly disordered carbon systems; percolation and graph zeta function

1. Activation carbon fiber
2. Conductivity of ACFs
   Kuriyama's Investigation
3. **Conductivity of percolation**
4. Graph Theory
5. New proposals on the conductivity
6. Summary

What is percolation

Percolation is a mathematical model in statistics and statistical mechanics.

Related to Fields Medal:

Wendelin Werner 2006

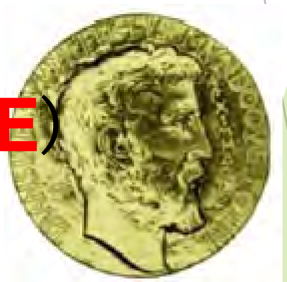
Stanislav Smirnov 2010

Schramm-Lawner-evolution（SLE）

・Oded Schramm (1961-2008)

・Lawner treatment of Bieberbach conjecture

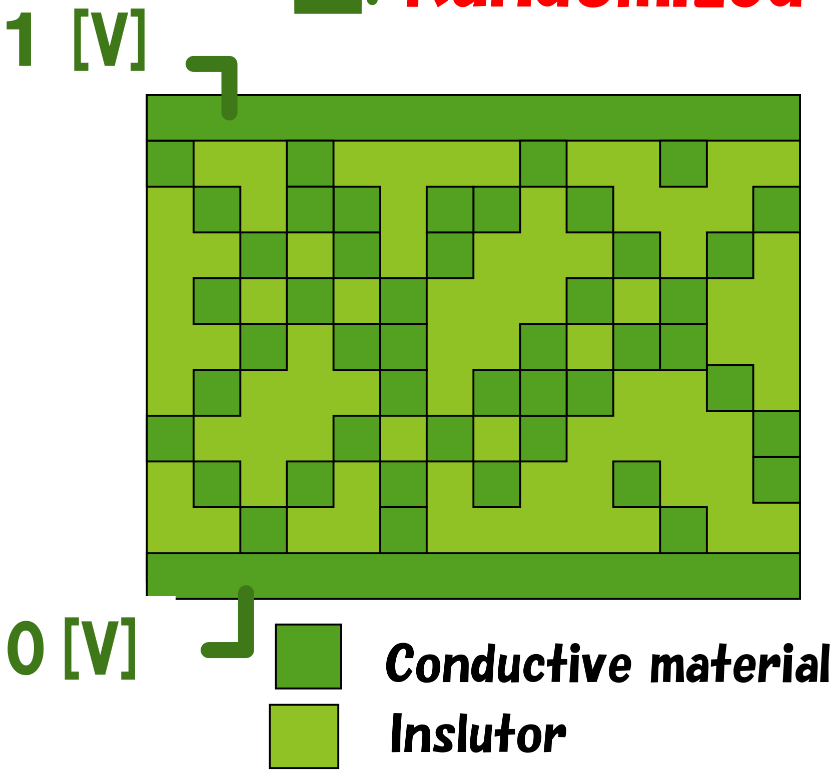
Robert Langlands, John Cardy

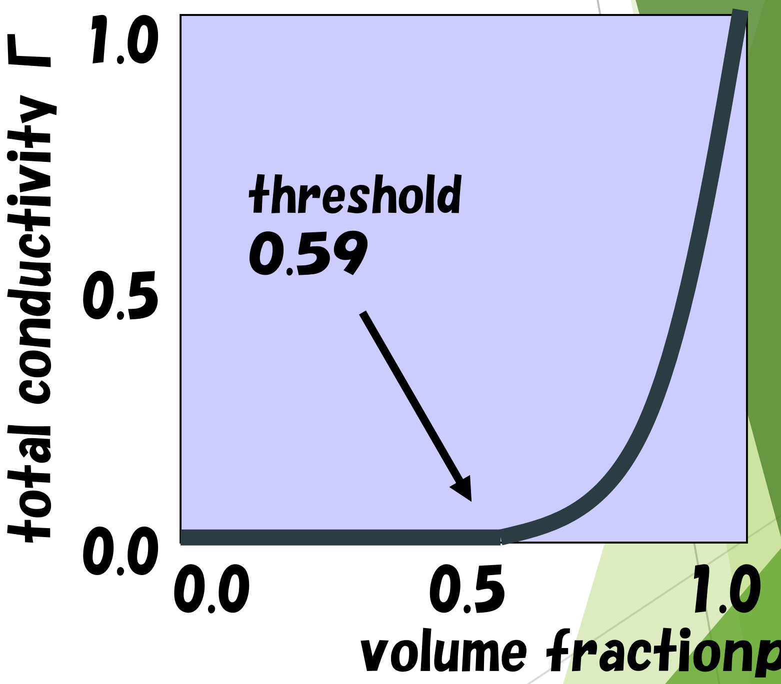


PERCOLATION

Lattice Percolation Model

■: Randomized Conductive materials



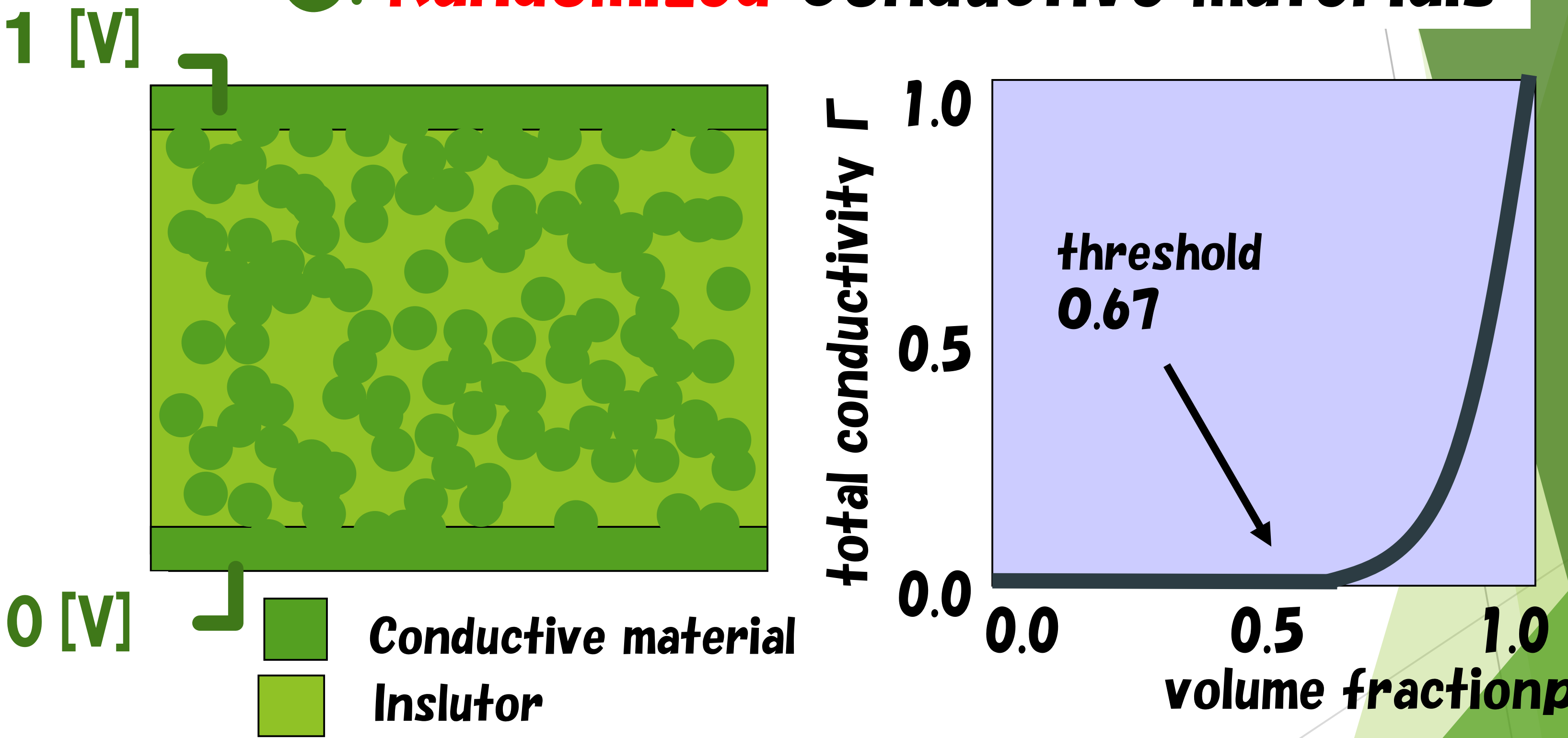


For infinite region, the total conductivity depends only on volume fraction(density of ■)

PERCOLATION

CONTINUUM PERCOLATION MODEL

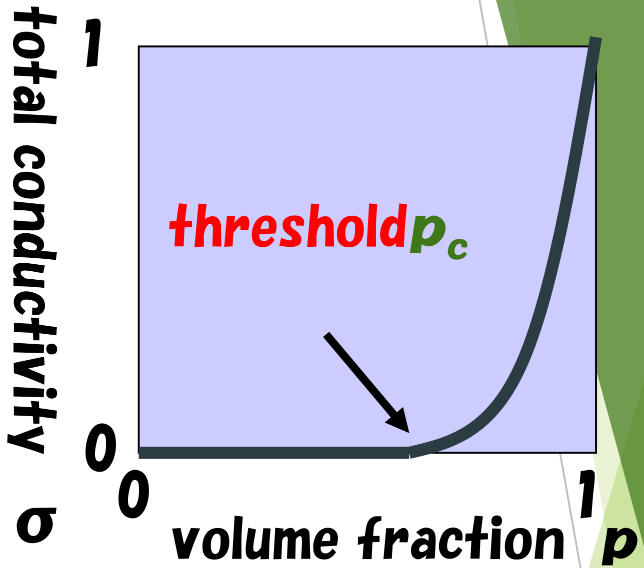
●: Randomized Conductive materials



For infinite region, the total conductivity depends only on volume fraction(density of ●)

Properties of percolation

- Depending on dimension
- Insulator $p <$ threshold p_c
- Conductor $p >$ threshold p_c
- Total conductivity drastically depends on p



$$\sigma = (p \quad p_c)^t \quad \Rightarrow \quad R = \frac{1}{(p - \ p_c)^t}$$

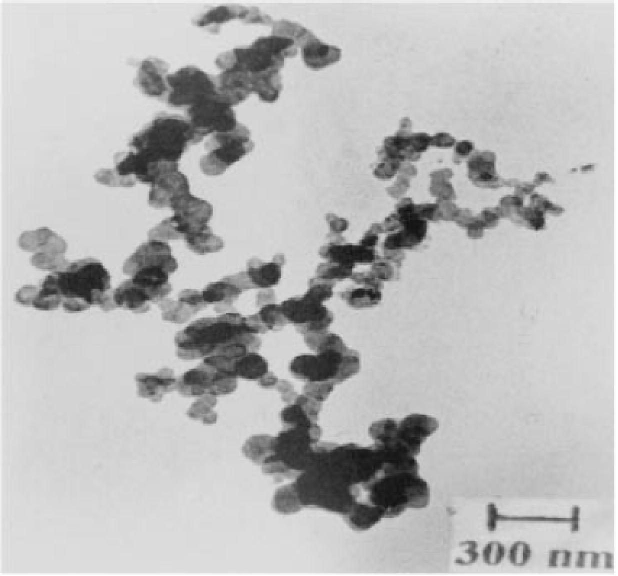
$p \geqq p_c$

$t > 0$

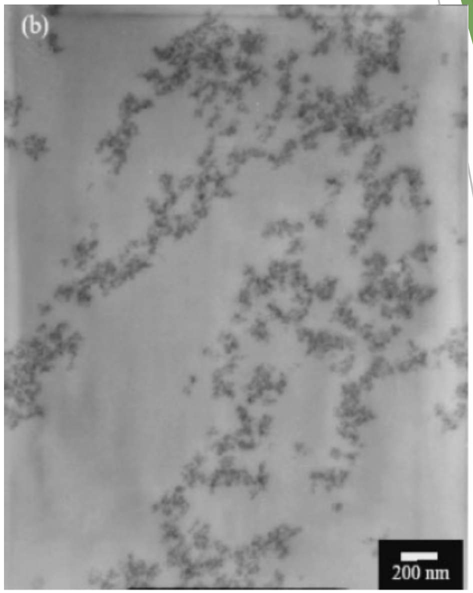
$R \to \infty$ at $p \sim p_c$

Conductive nano materials in insulator

Examples



Preparation and Some Properties of a Nanocomposite of Polyacrylonitrile with Acetylene Black Arjun Maity and Mukul Biswas Polymer Journal, Vol. 36 (2004) No. 10 pp.812-816



Fine Dispersion and Property Differentiation of Nanoscale Silicate Platelets and Spheres in Epoxy Nanocomposites Chien-Chia Chu, Jiang-Jen Lin, Chang-Ru Shiu and Chang-Chin Kwan Polymer Journal, Vol. 37 (2005) No. 4 pp.239-245

Real Material

「Insulator = material with high resistance

「Conductive material = material with small resistance

Control total conductivity by percolation theory

Two dimensional continuum percolation model

1. Put conductive particles with unit radius into high-resistance material randomly

2. Set the electrodes and apply the voltage

3. With Dirichlet-Neumann B.C. Solve the equation

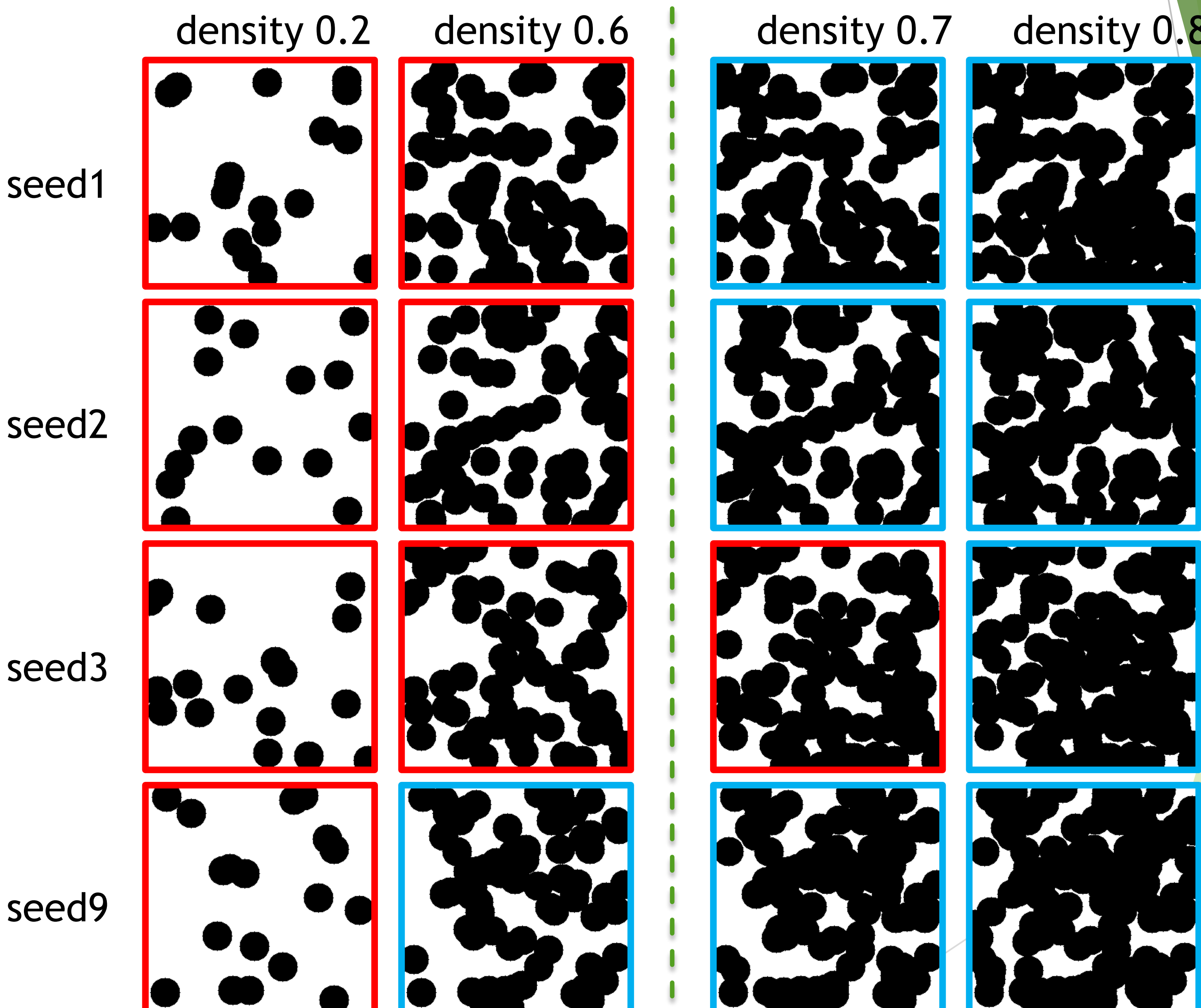
$$\mathrm{div}(\gamma \cdot \mathrm{grad}\ u) = 0$$

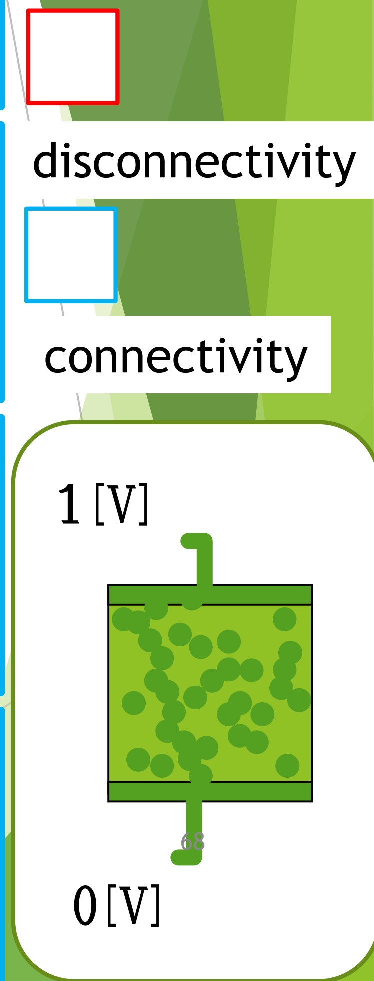
● conductive particle $\gamma(x) = 1$

□ high-resistance material $\gamma(x) = 10^{-4}$

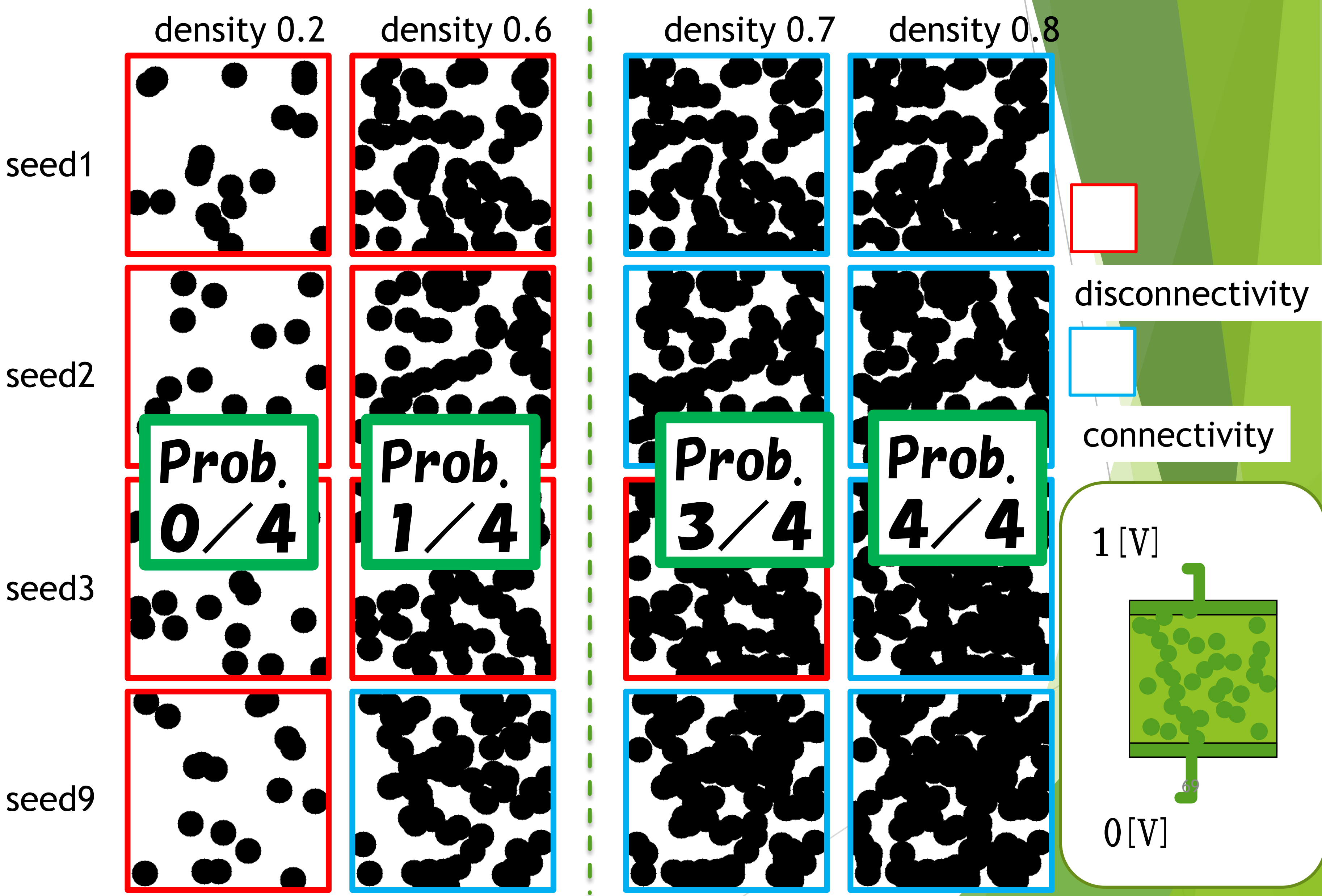
Matsutani, Shimosako, Wang
Physica A **391** (2012) 5802-5809
Fractal Structure of Equipotential Curves on a Continuum Percolation Model

Ordinary continuum percolation model

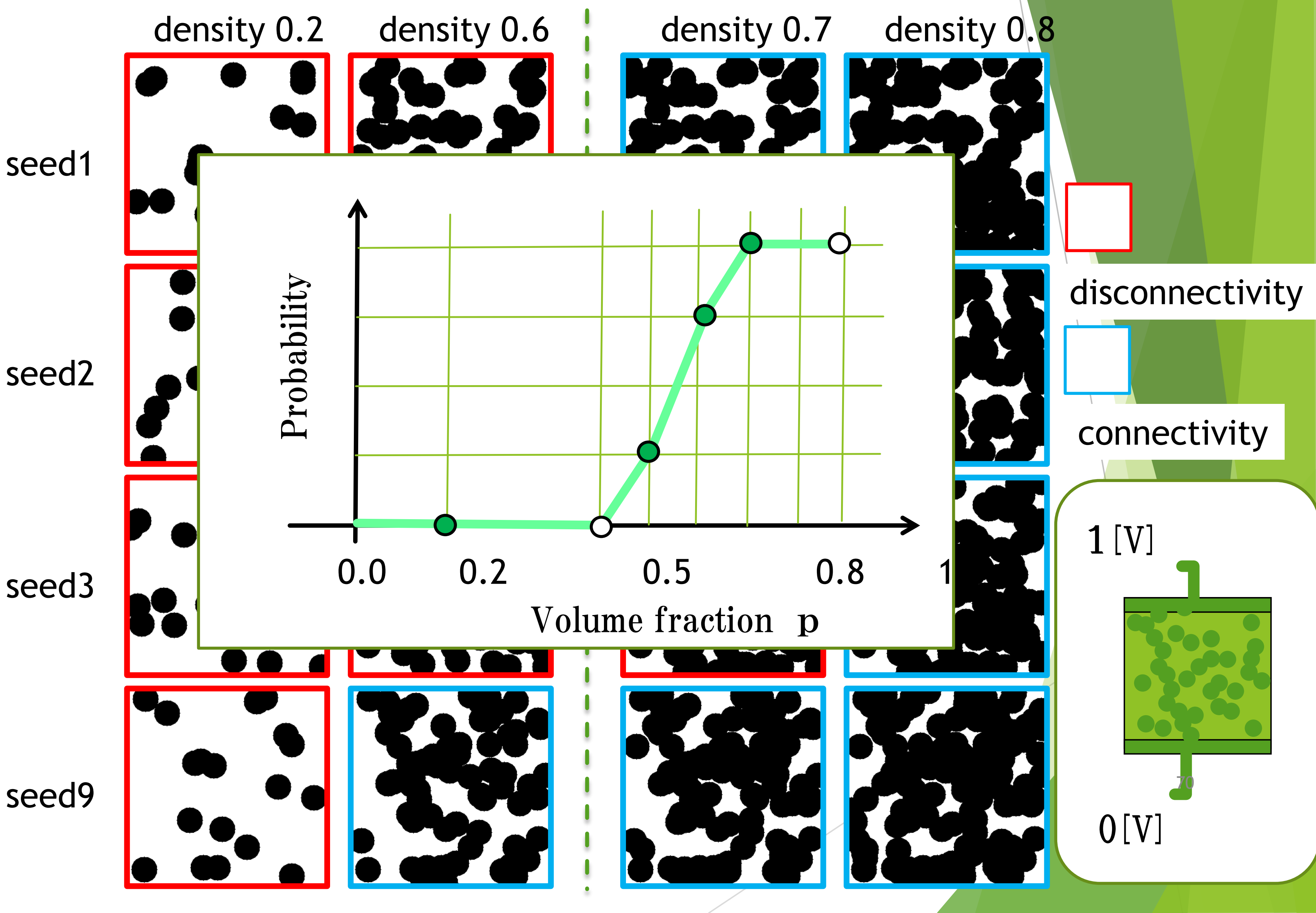




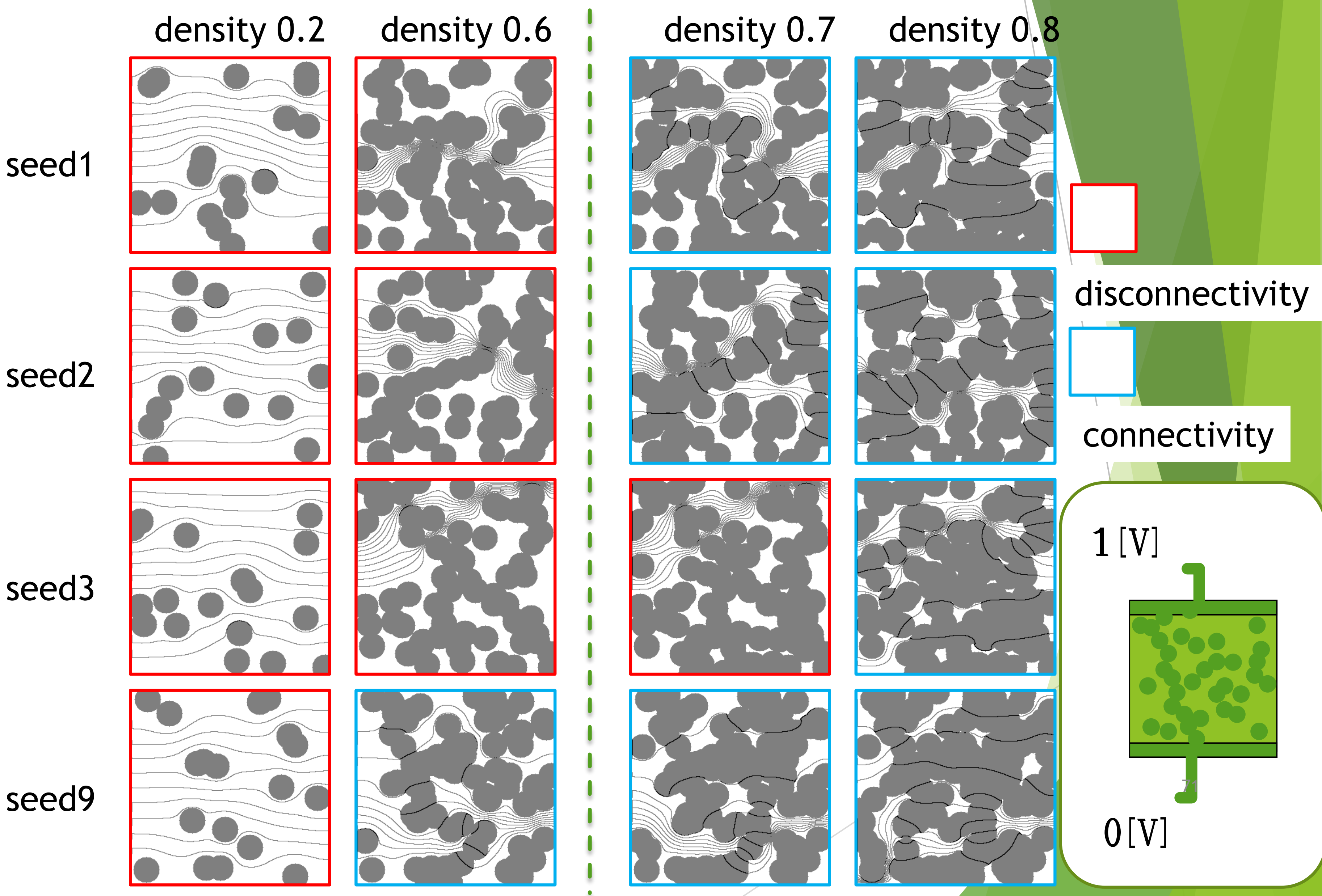
Ordinary continuum percolation model



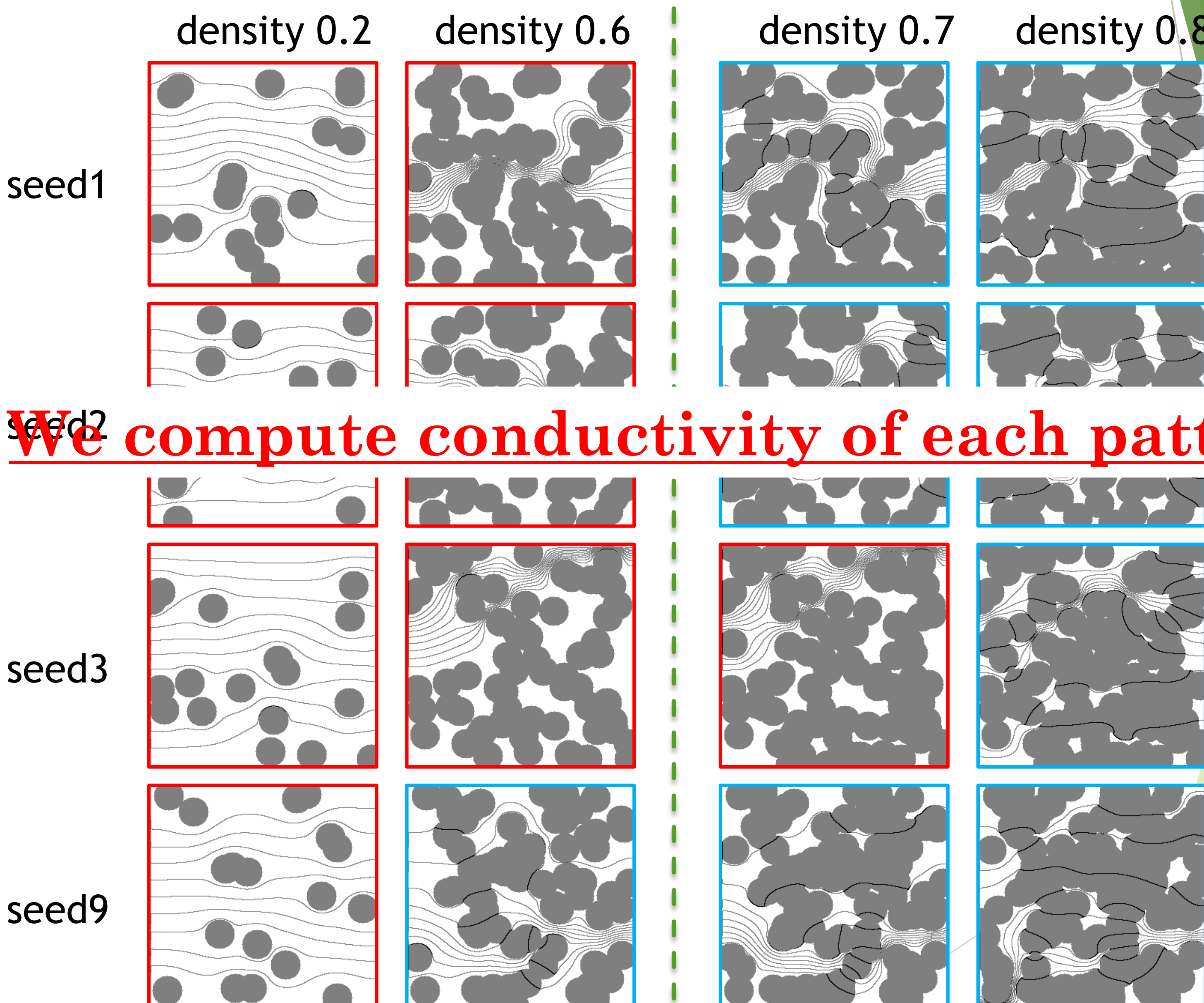
Ordinary continuum percolation model



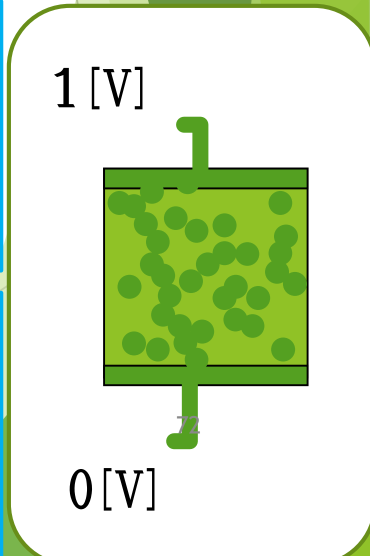
Conductivity of continuum percolation model



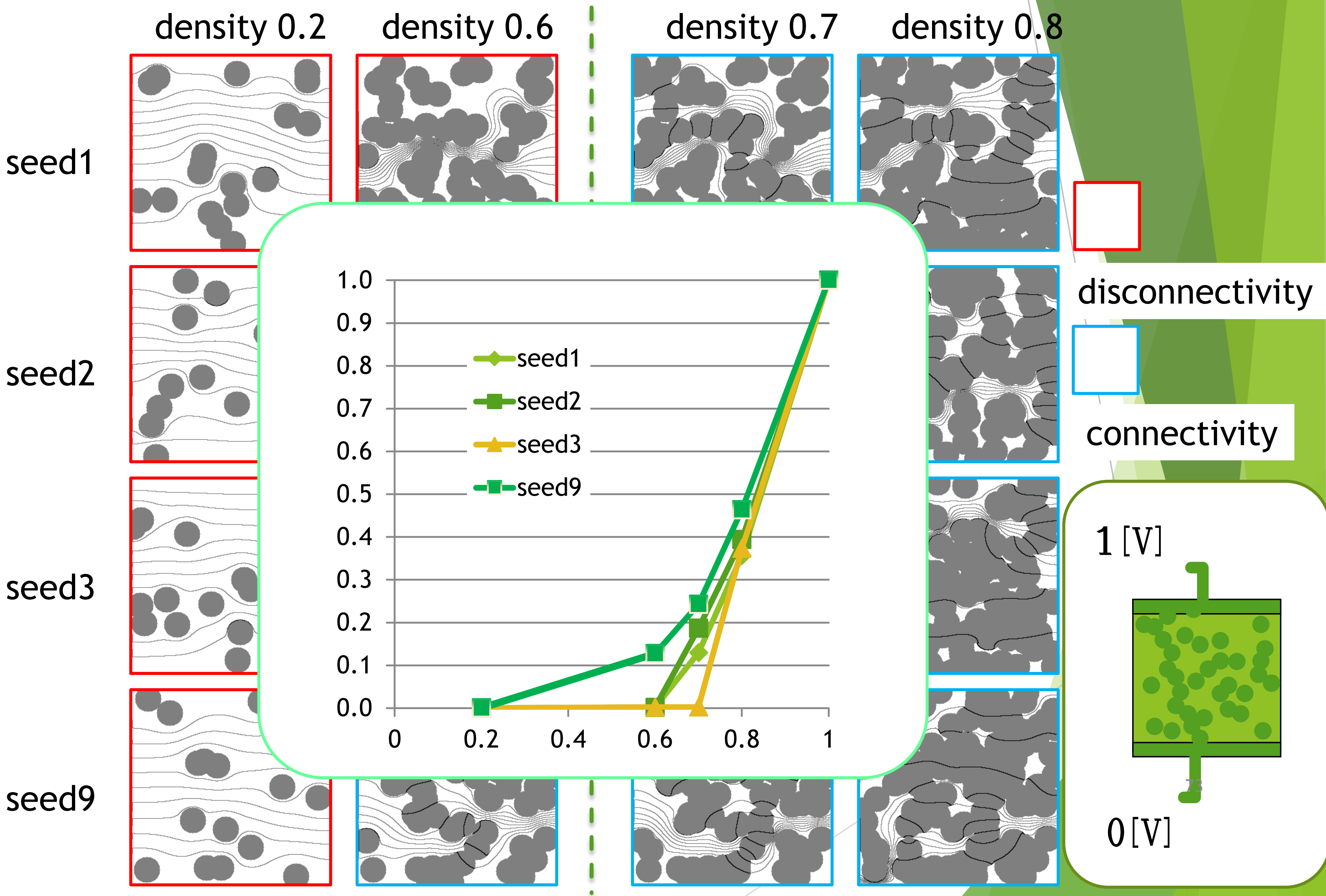
Conductivity of continuum percolation model



We compute conductivity of each pattern

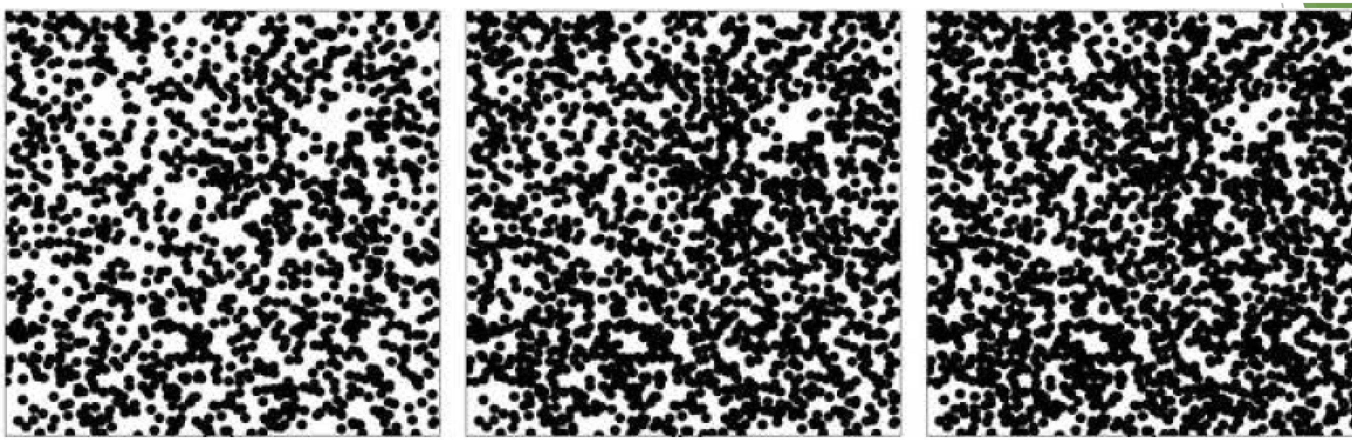


Conductivity of continuum percolation model

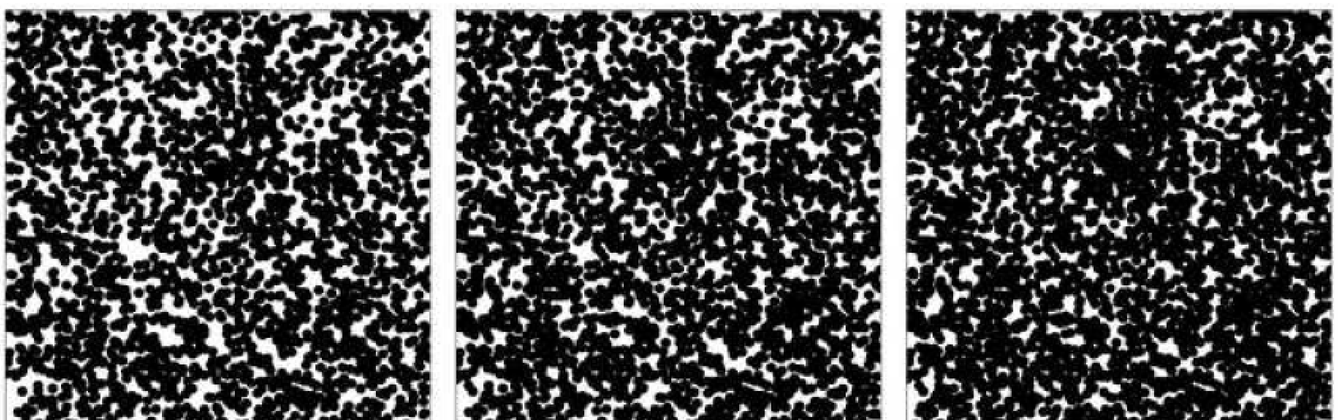


Two dimensional continuum percolation model

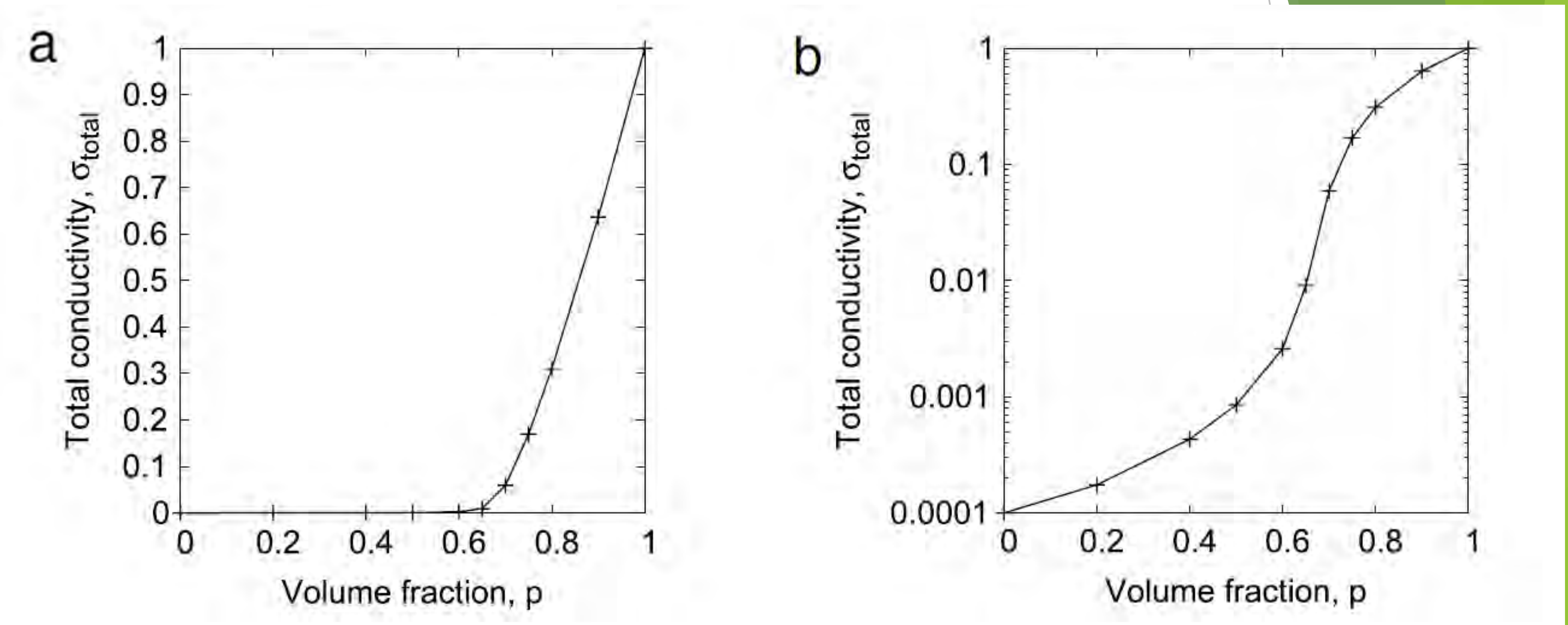
$p_c = 0.67$



volume fraction p=0.5　　0.6　　0.65



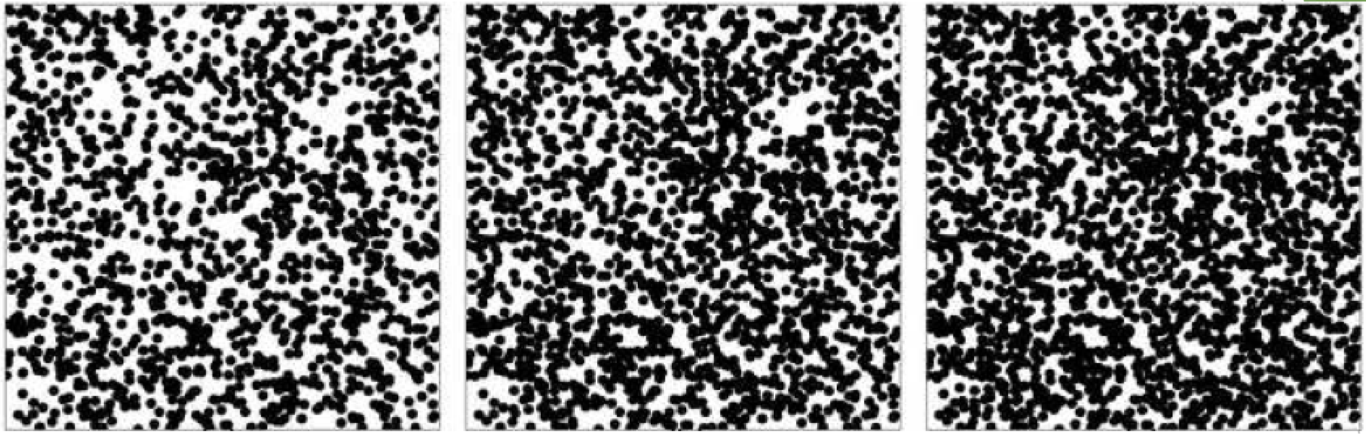
volume fraction p=0.7　　0.75　　0.8



M-Shimosako-Wang,
IJCPC 2010, Physica A 2012,
AMM 2013, 2015

Two dimensional continuum percolation model

$p_c = 0.67$



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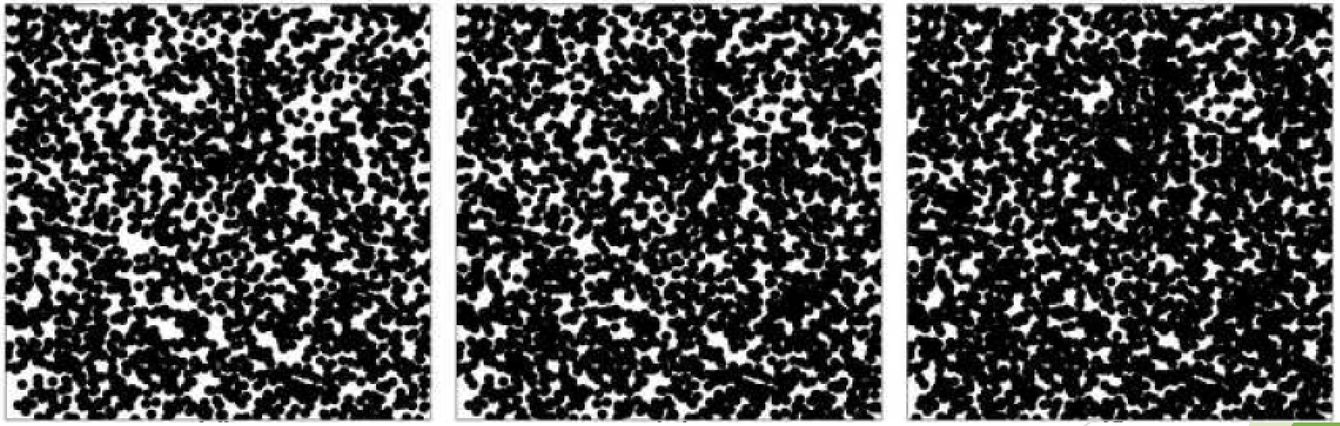
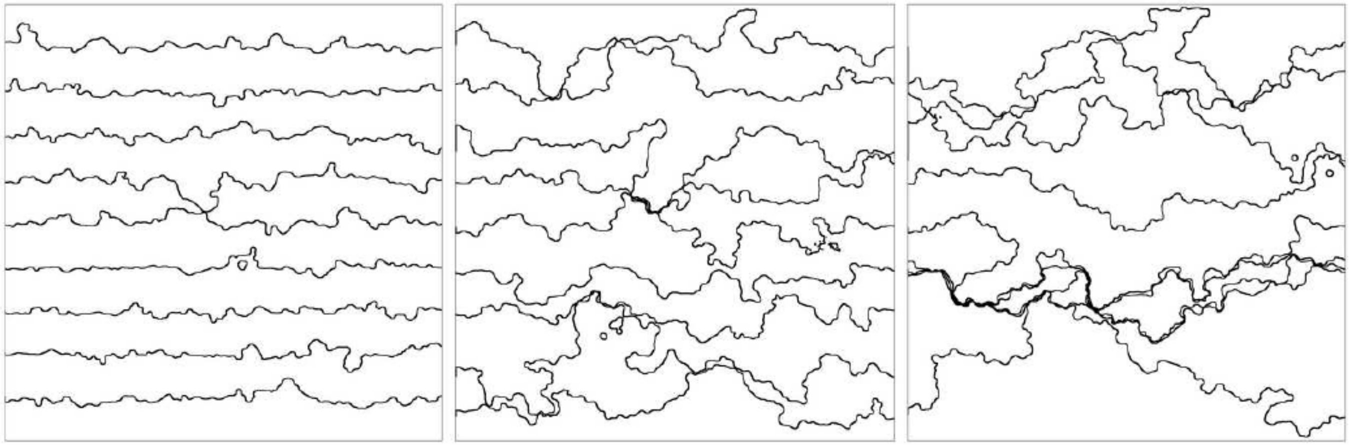


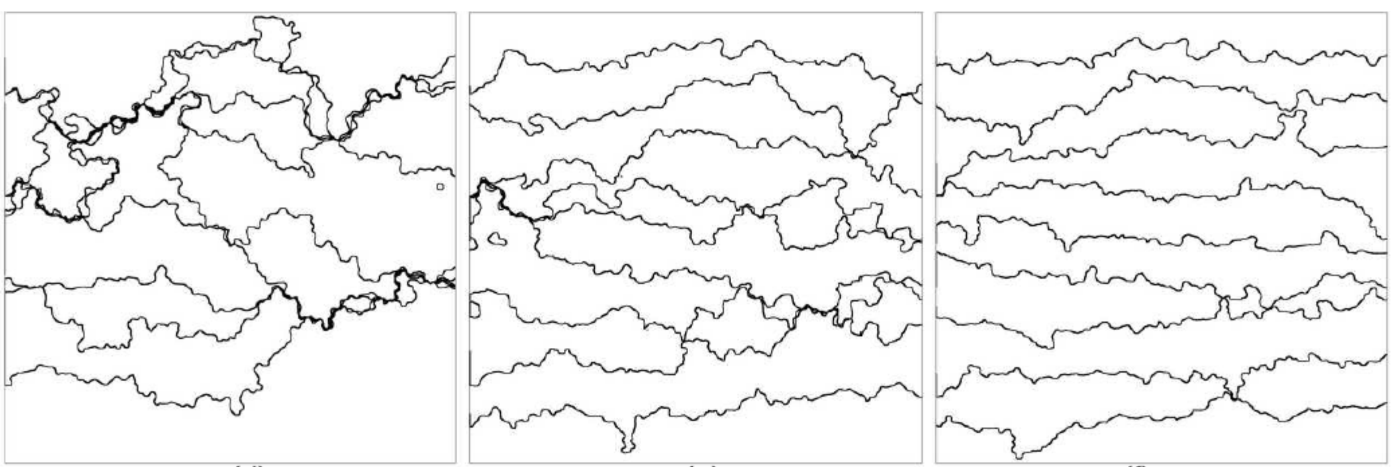
volume fraction p = 0.7 0.75 0.8

Equi-potential curves

$p_c = 0.67$

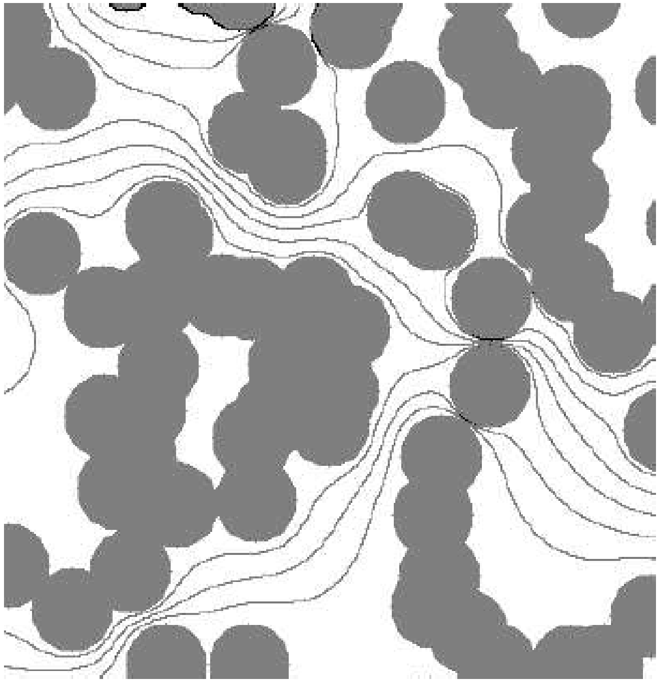


volume fraction p = 0.5 0.6 0.65

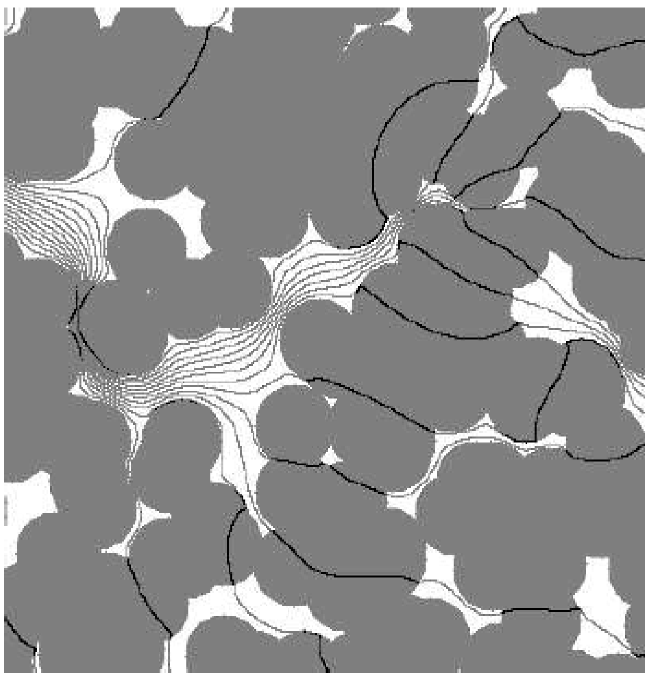


volume fraction p = 0.7 0.75 0.8

equi-potential curves



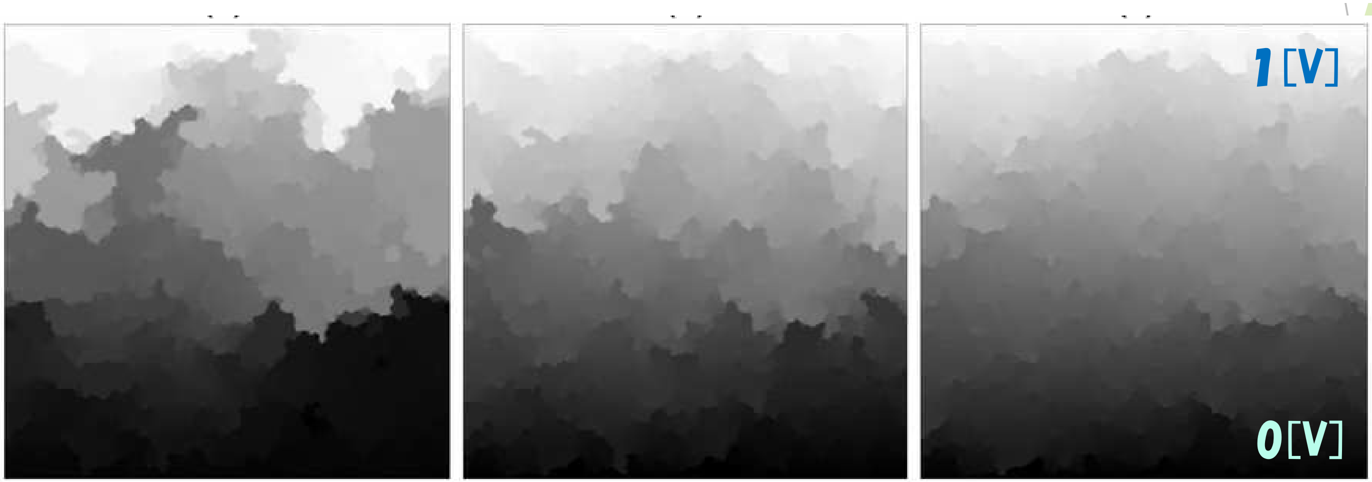
volume fractionp=0.5



volume fractionp=0.8

electric potential dist. $p_c = 0.67$

volume fraction p = 0.5　　0.6　　0.65

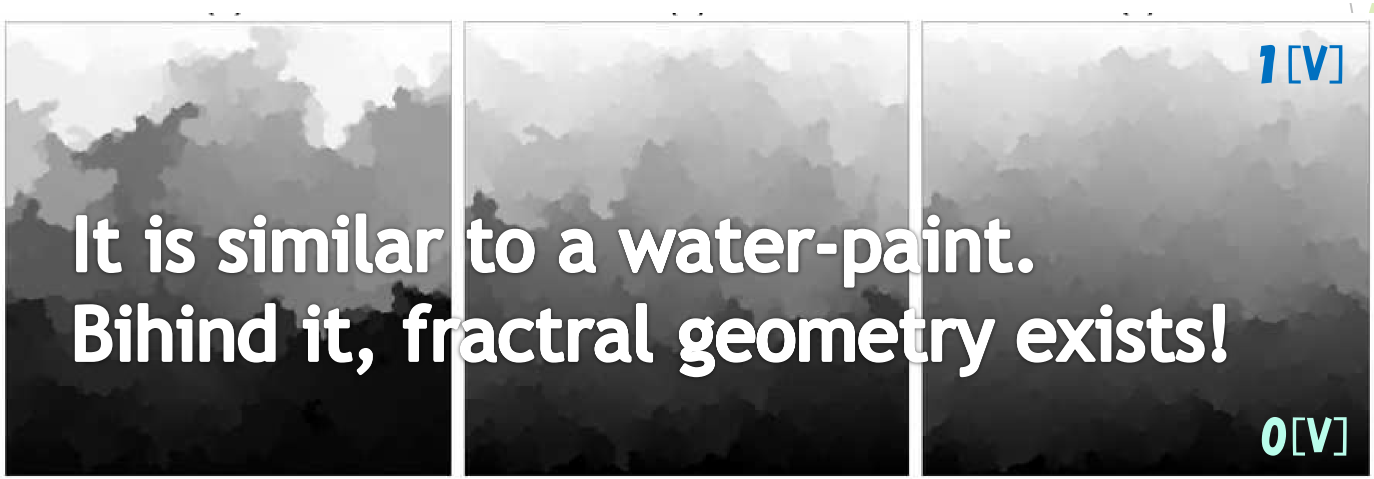


volume fraction p = 0.7　　0.75　　0.8

electric potential dist.

$p_c = 0.67$

volume fraction p=0.5 0.6 0.65

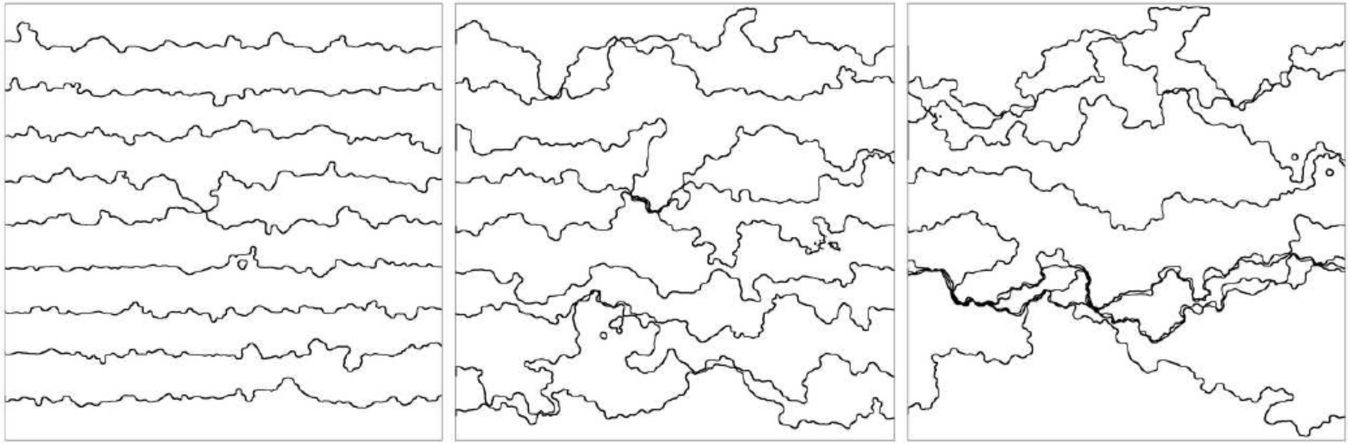


It is similar to a water-paint.
Bihind it, fractral geometry exists!

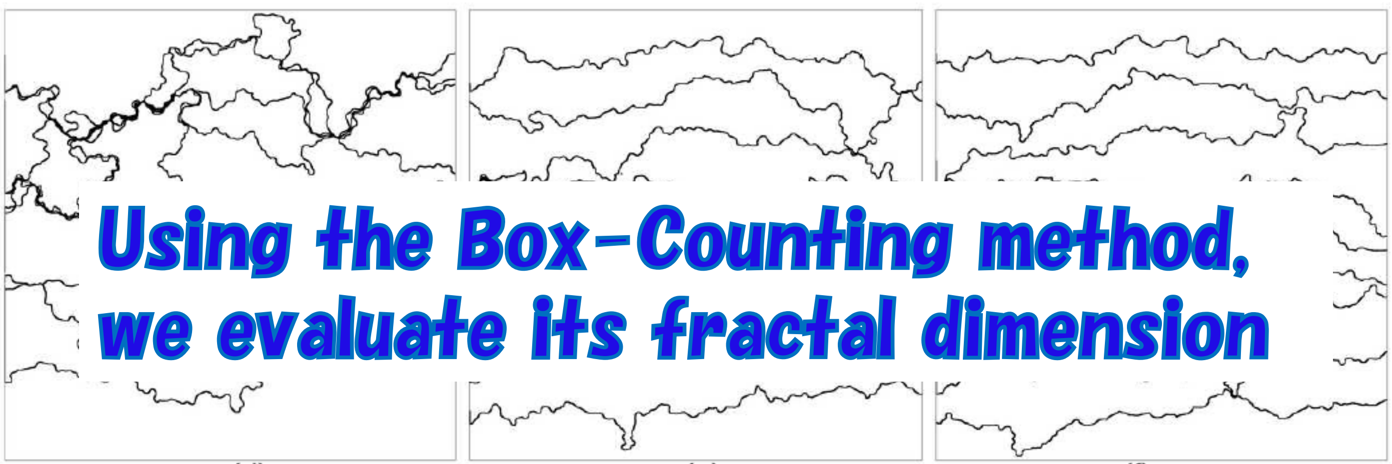
volume fraction p=0.7 0.75 0.8

equi-potential curves

$p_c=0.67$

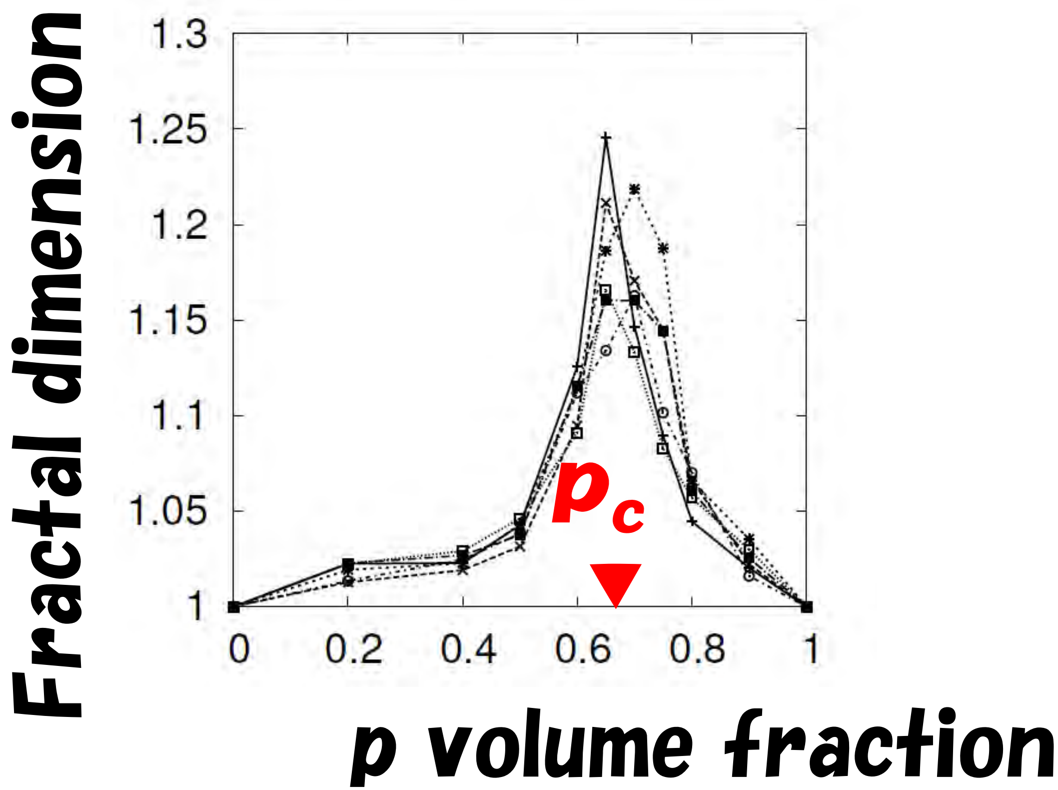


volume fraction p = 0.5 0.6 0.65



Using the Box-Counting method, we evaluate its fractal dimension

volume fraction p = 0.7 0.75 0.8

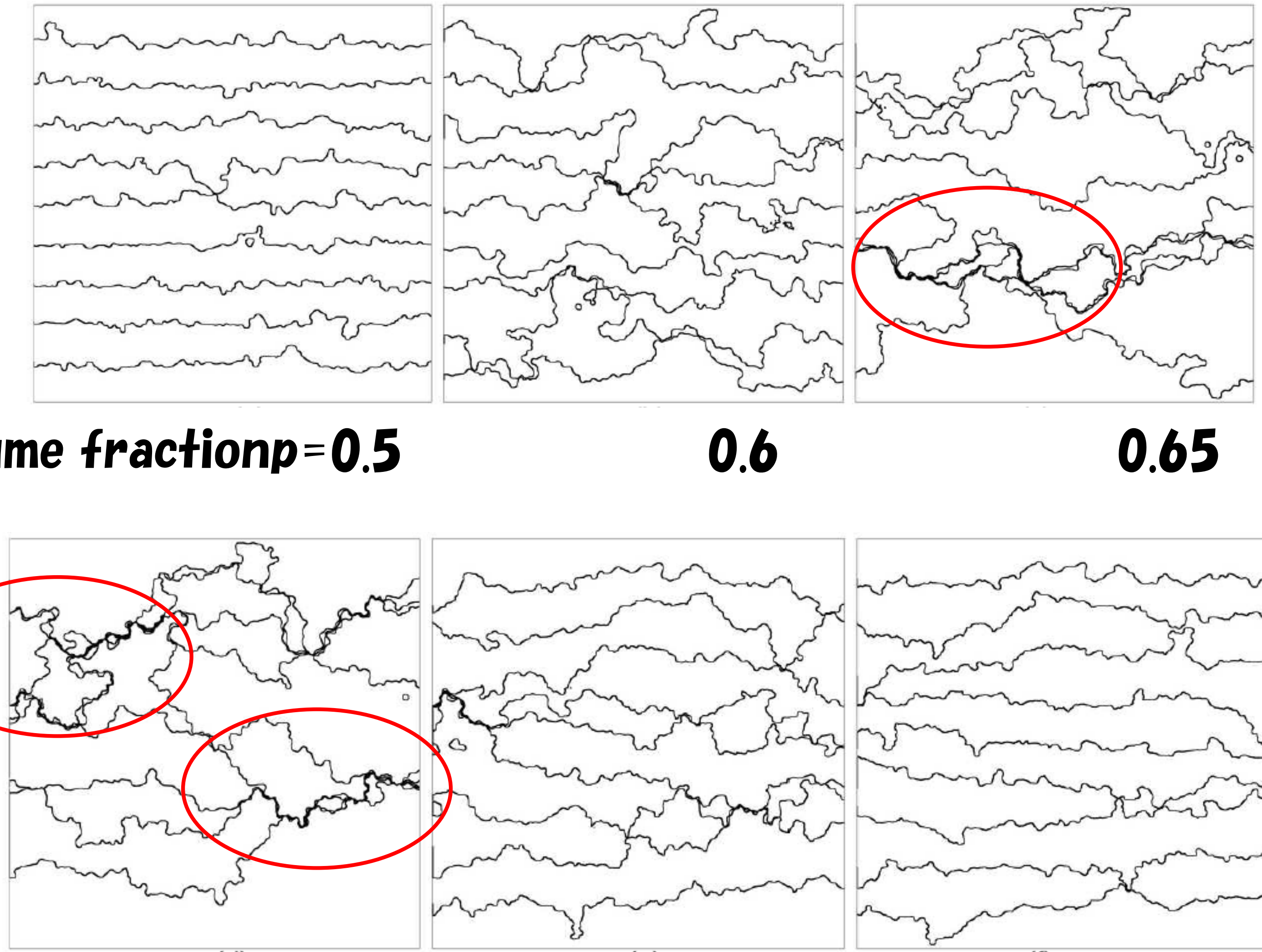


Around p_c, the electric potential distribution shows fractal geometry.

M-Shimosako-Wang, Physica A 2012,

equi-potential curves

$p_c = 0.67$

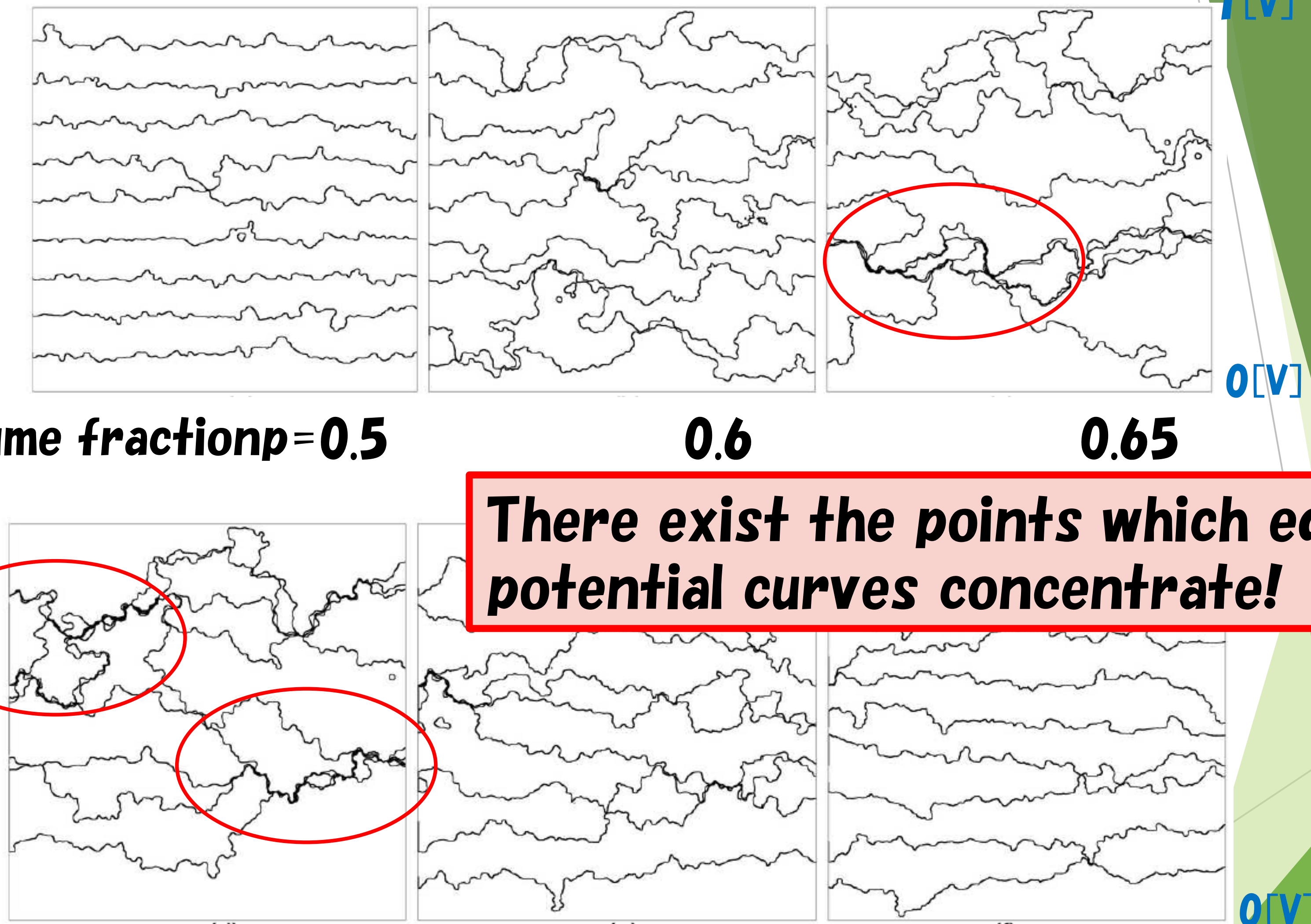


volume fraction p = 0.5 0.6 0.65

volume fraction p = 0.7 0.75 0.8

equi-potential curves

$p_c = 0.67$



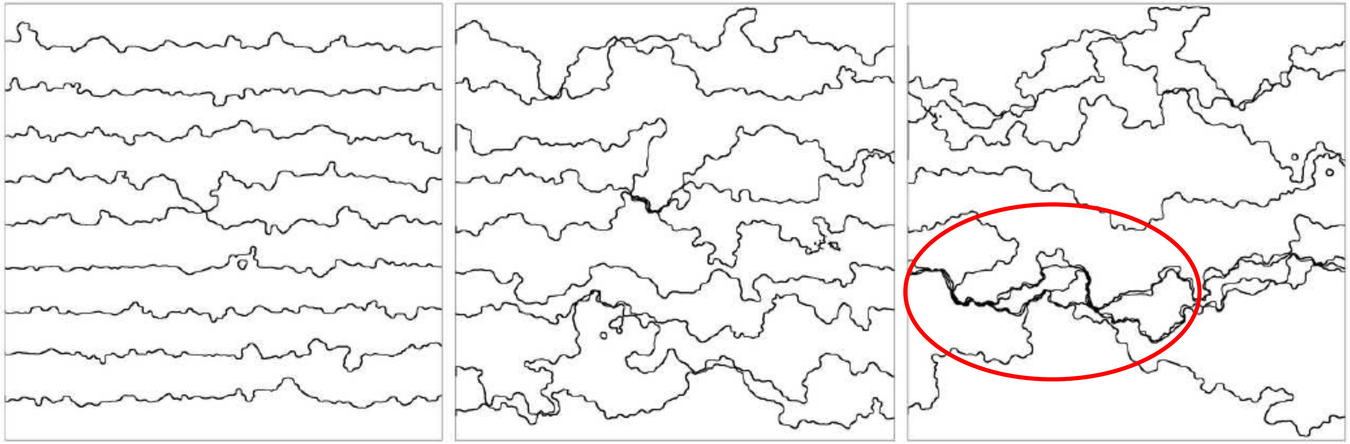
volume fractionp=0.5 0.6 0.65

There exist the points which equi-potential curves concentrate!

volume fractionp=0.7 0.75 0.8

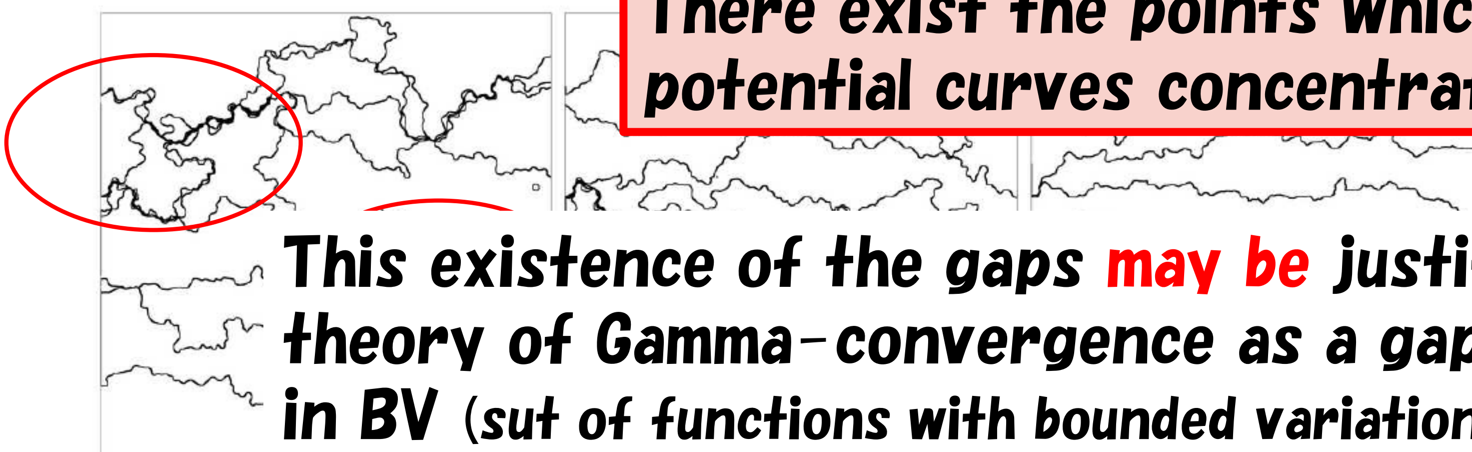
equi-potential curves

$p_c = 0.67$



volume fractionp=0.5 0.6 0.65

There exist the points which equi-potential curves concentrate!

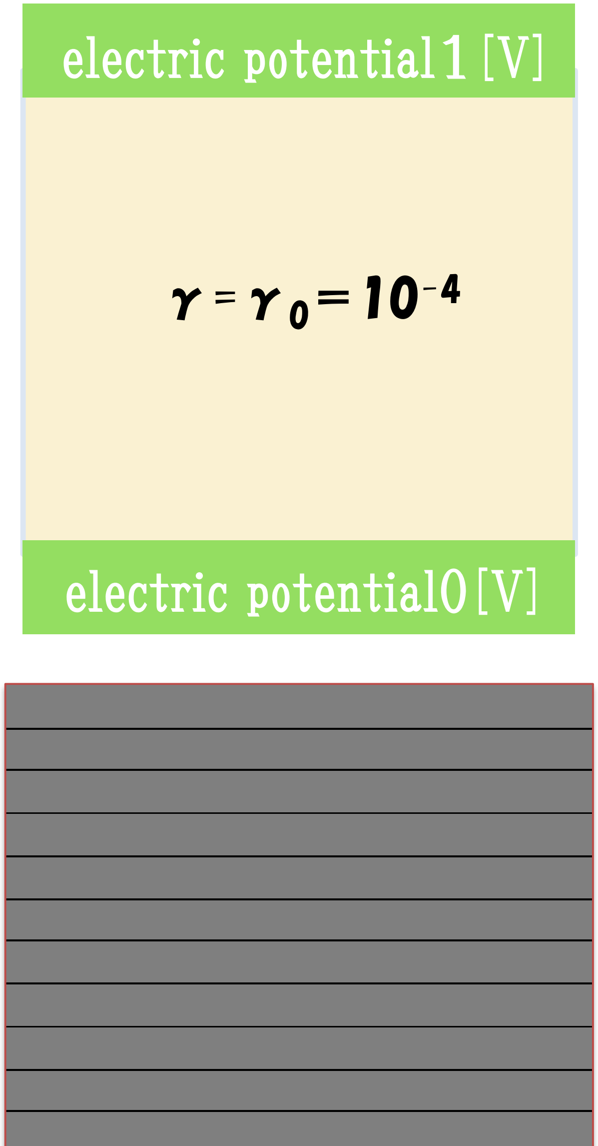


This existence of the gaps may be justified the theory of Gamma-convergence as a gap solutions in BV (sut of functions with bounded variation).

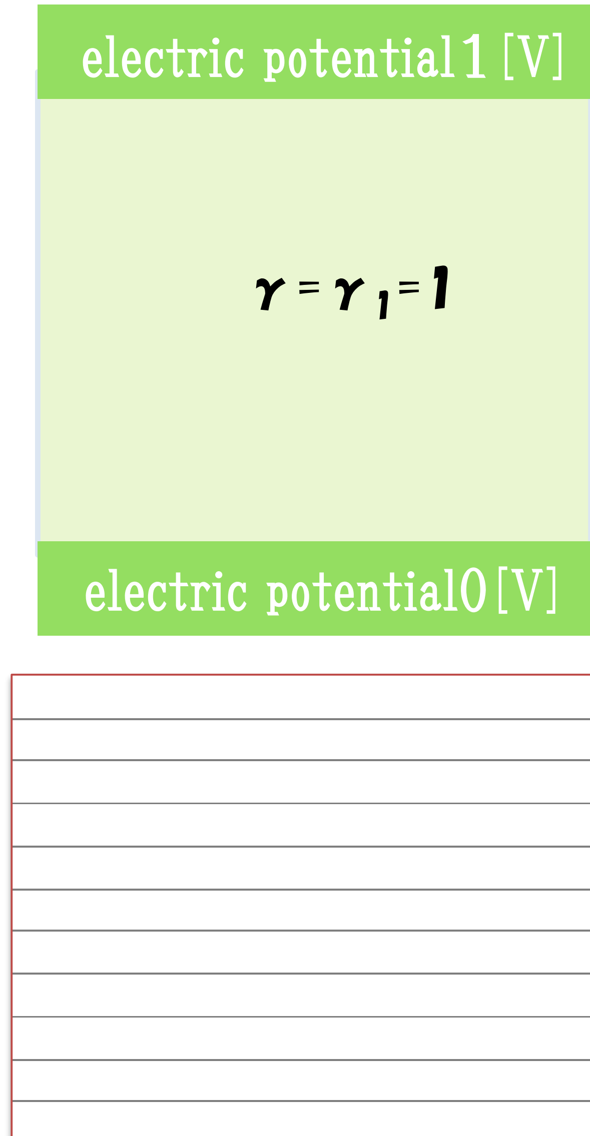
volume fractionp=0.7 0.75 0.8

Duality in Electric Potential Distribution
(Keller-Dykhne Duality)

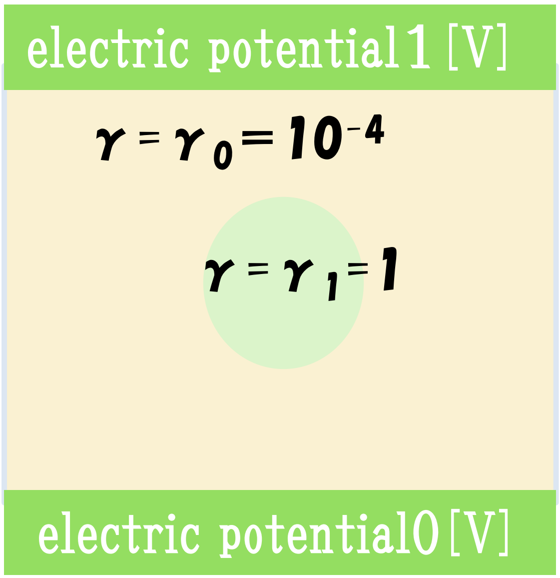
γ : insulator

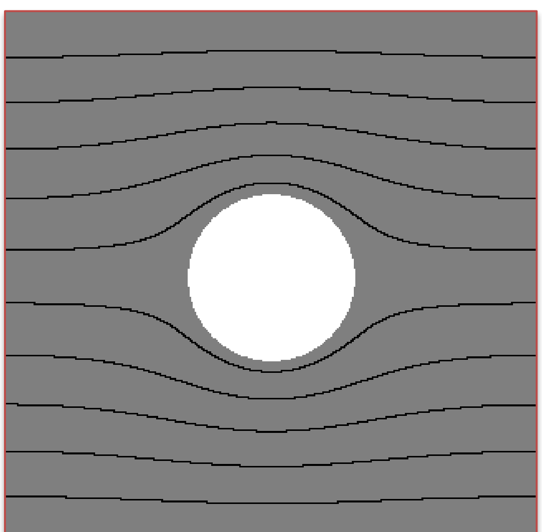


γ : conductor

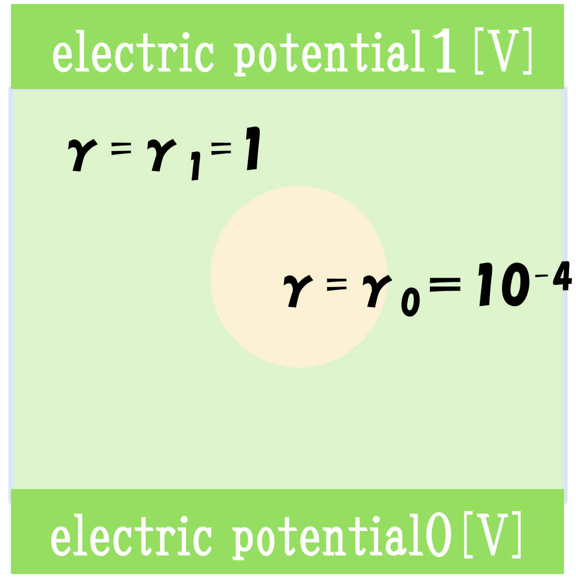


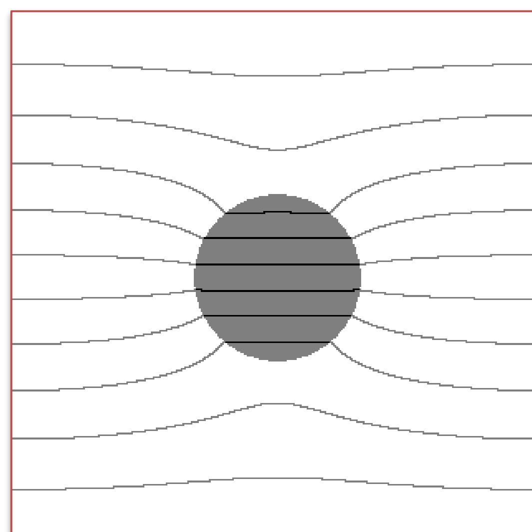
γ :Conductor ／insulator

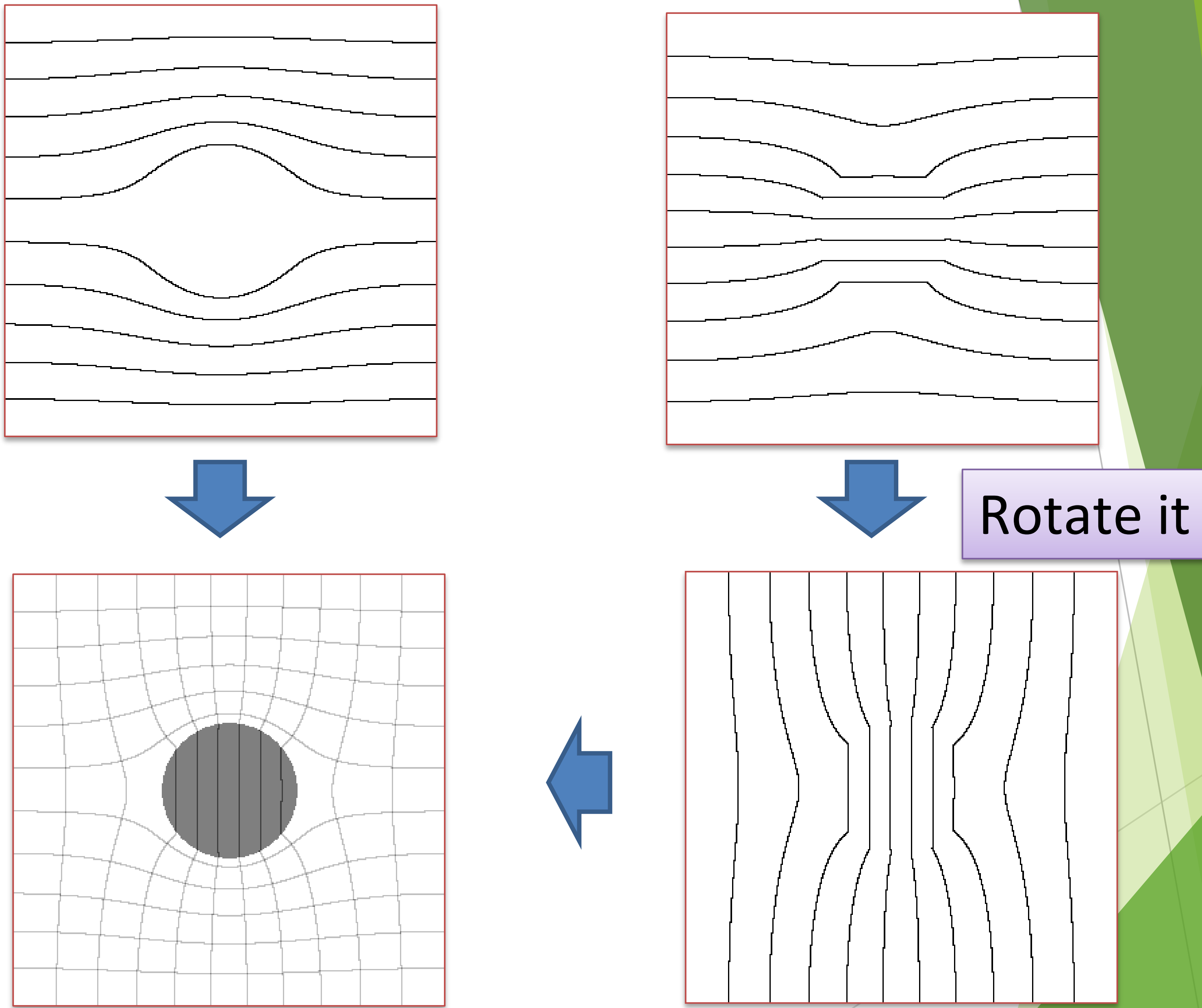




γ :insulator ／Conductor

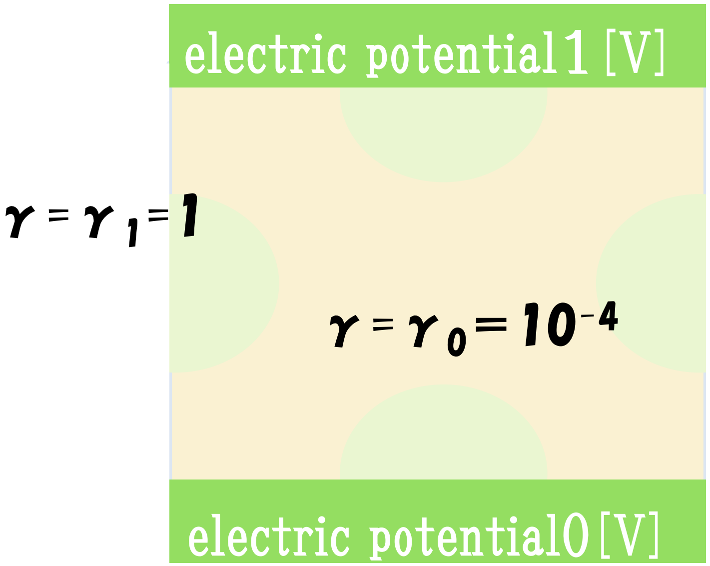


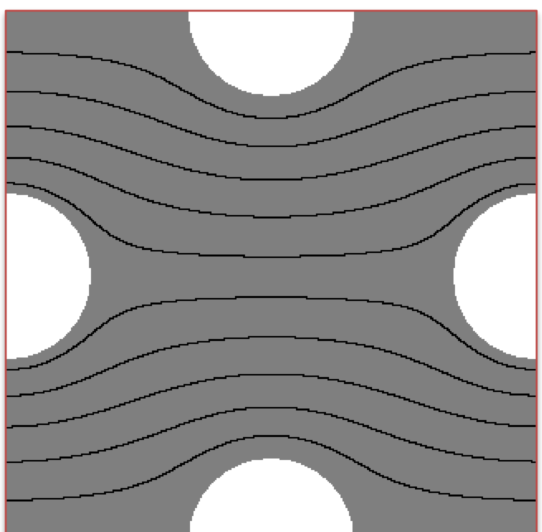




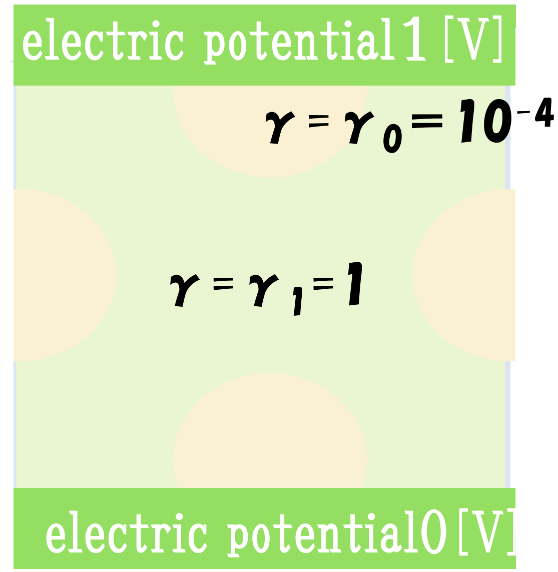
Perpendicular

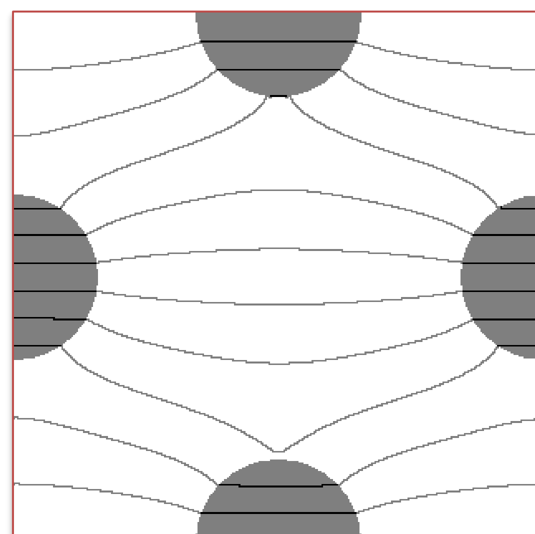
γ : Conductor / insulator

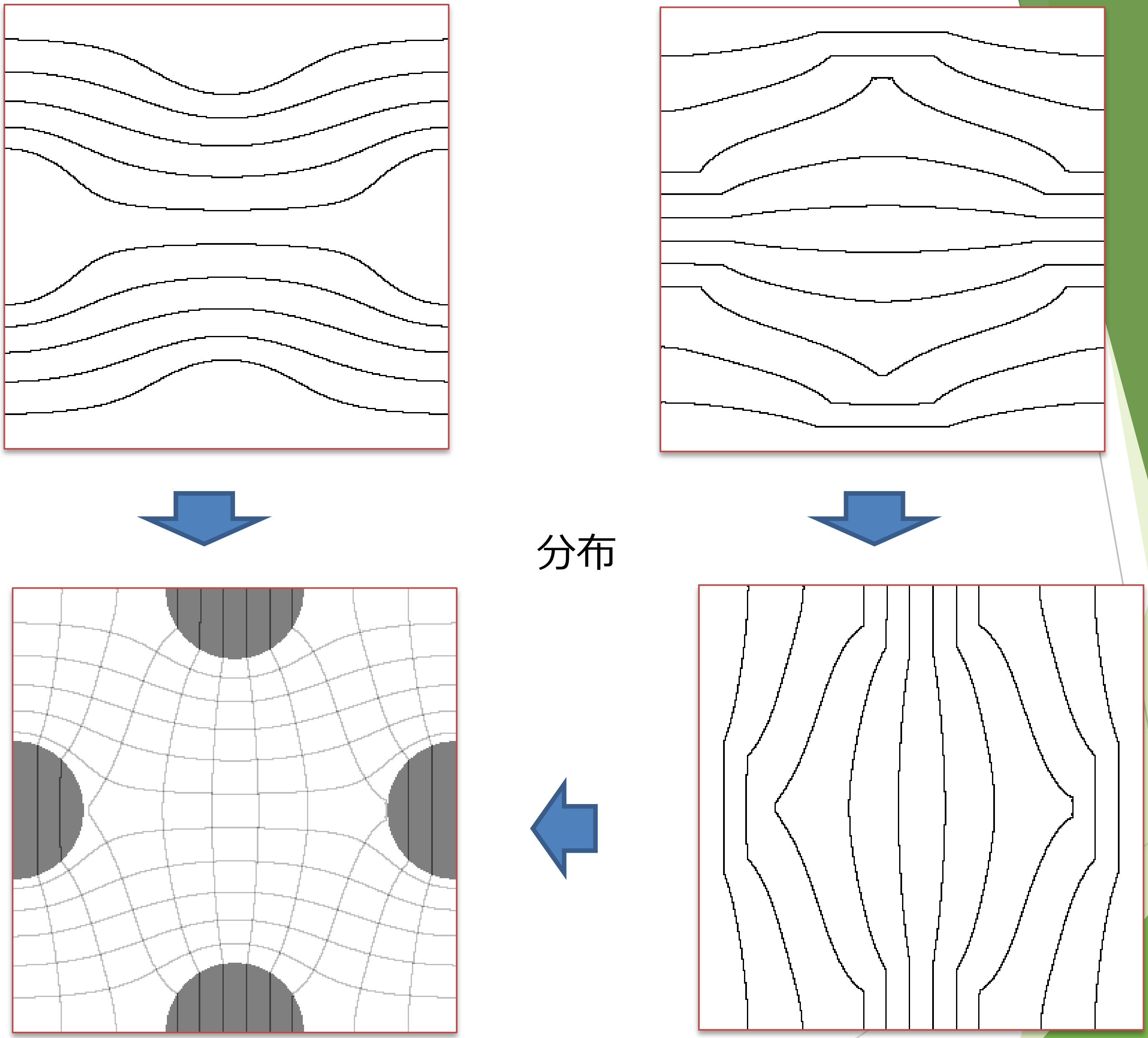




γ : insulator / Conductor

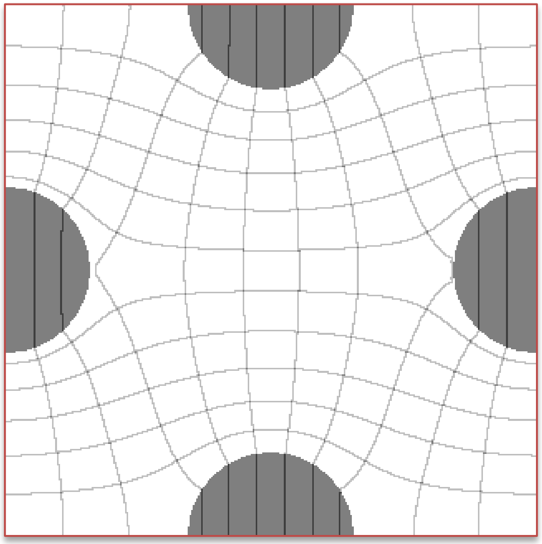


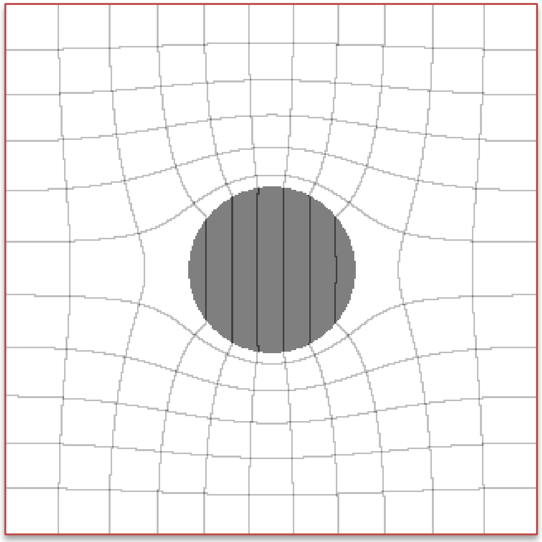




Perpendicular

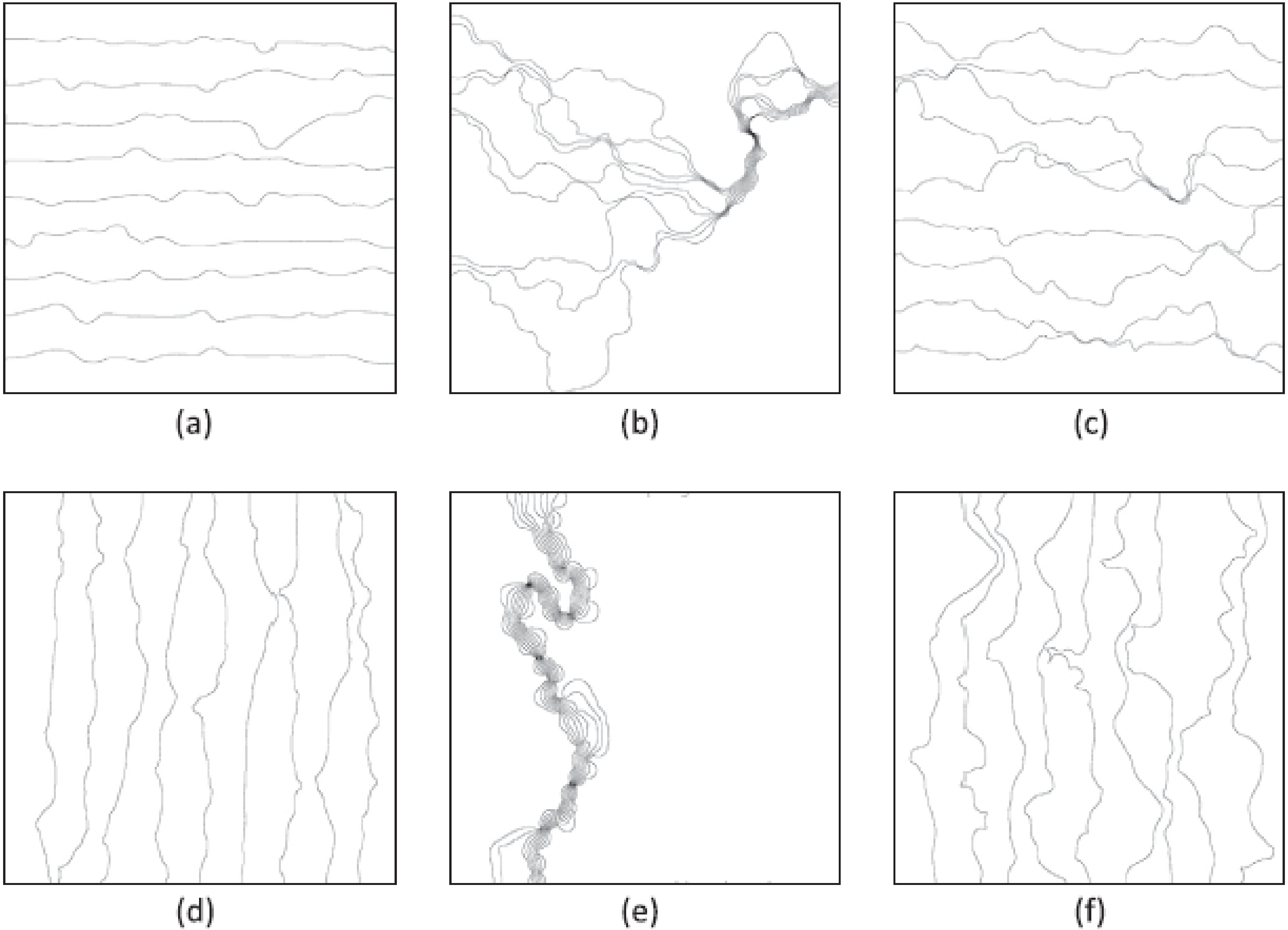
There is a duality!





Perpendicular

Due to the duality, current paths concentrate

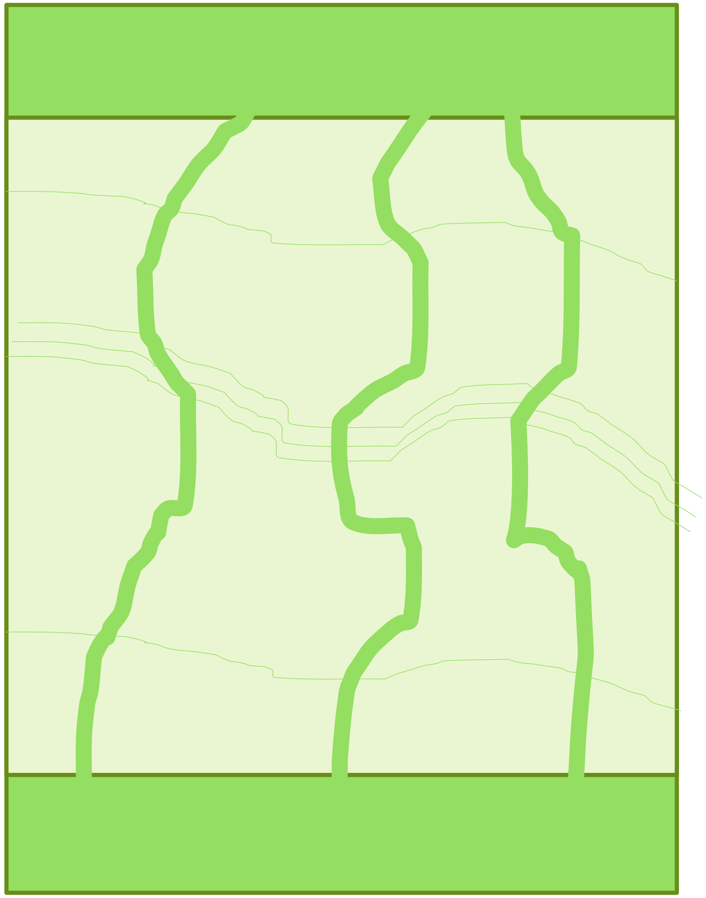


In randomized conductive system, there are places which have the concentrated electric field.
Such existence is basically known as the jump solutions in Gamma-convergence method in analysis.

Due to the duality, these pictures show that in the randomized system
1) There are several conductive paths
2) In one of paths, there is a concentrated field strength or a highly resistance part.

M-Shimosako, AMM 2015

Picture of conductivity in randomized material.



1. There are conductive paths C_i, $(i=1,2,\cdots,n)$.
2. In each path, there is highly resistant place (gap)

Picture of conductivity in randomized material.

1. There are conductive paths C_i, $(i=1,2,\cdots,n)$.
2. In each path C_i which consists of local conductors σ_{ij}, the conductivity of each path is given as

$$\sigma_i = \left(\sum_j \frac{1}{\sigma_{ij}} \right)^{-1}$$

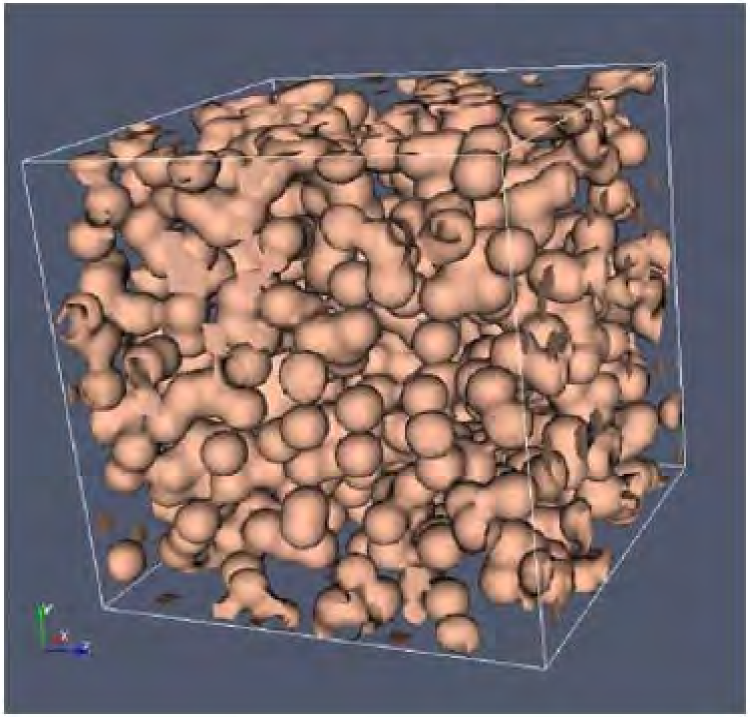
In each path, there is highly resistant place (gap)

$$\sigma_i \fallingdotseq \left(\max_j \frac{1}{\sigma_{ij}} \right)^{-1}$$

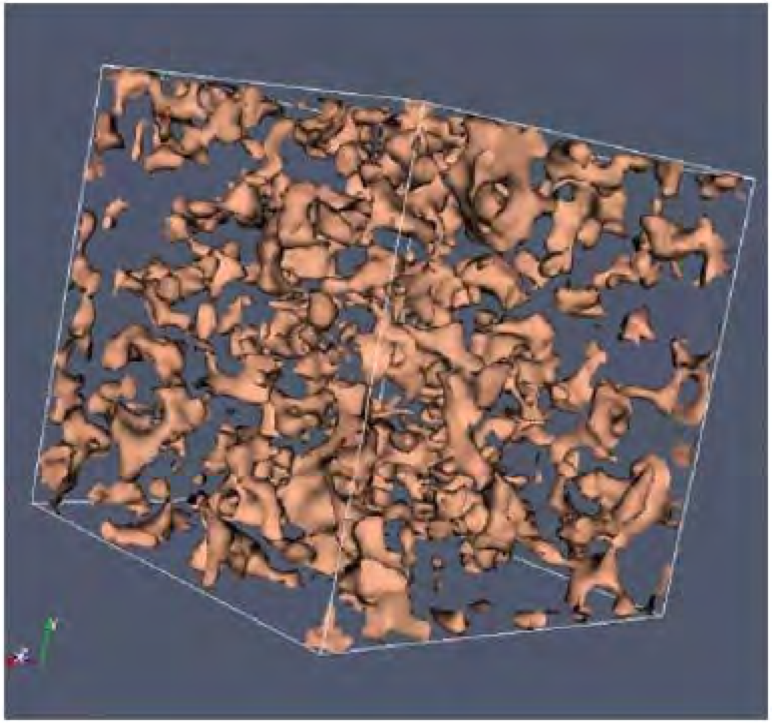
3. The total conductivity is given by,

$$\sigma_{total} \fallingdotseq \sum_i \min_j \sigma_{ij}$$

The picture basically is extended to three dimensional cases though the conductivity path is one-dimensional but equipotential surfaces are two-dimensional.



30%



90%

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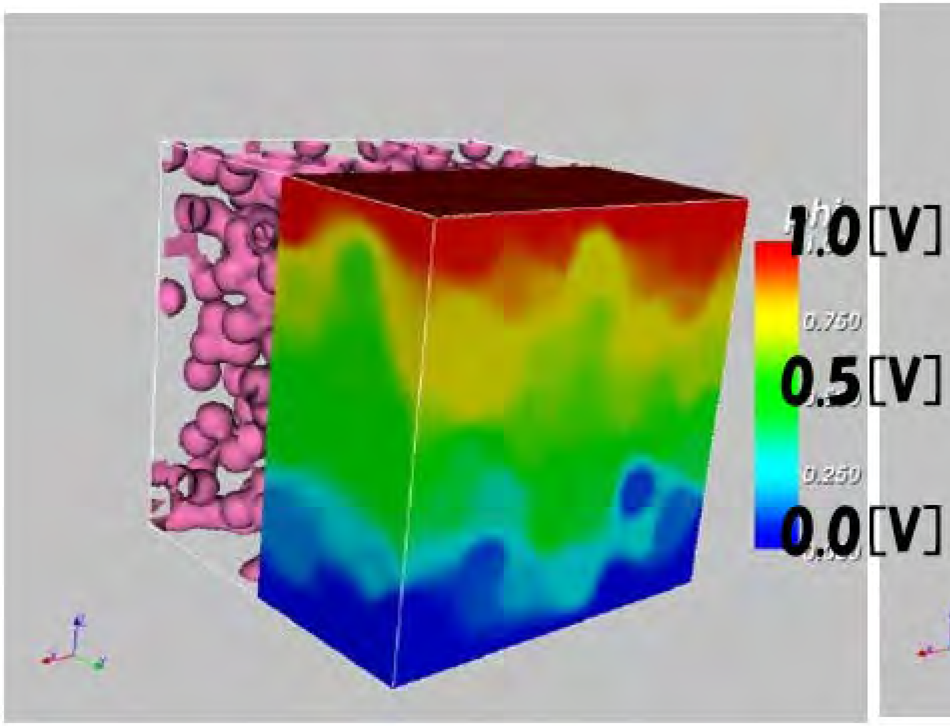
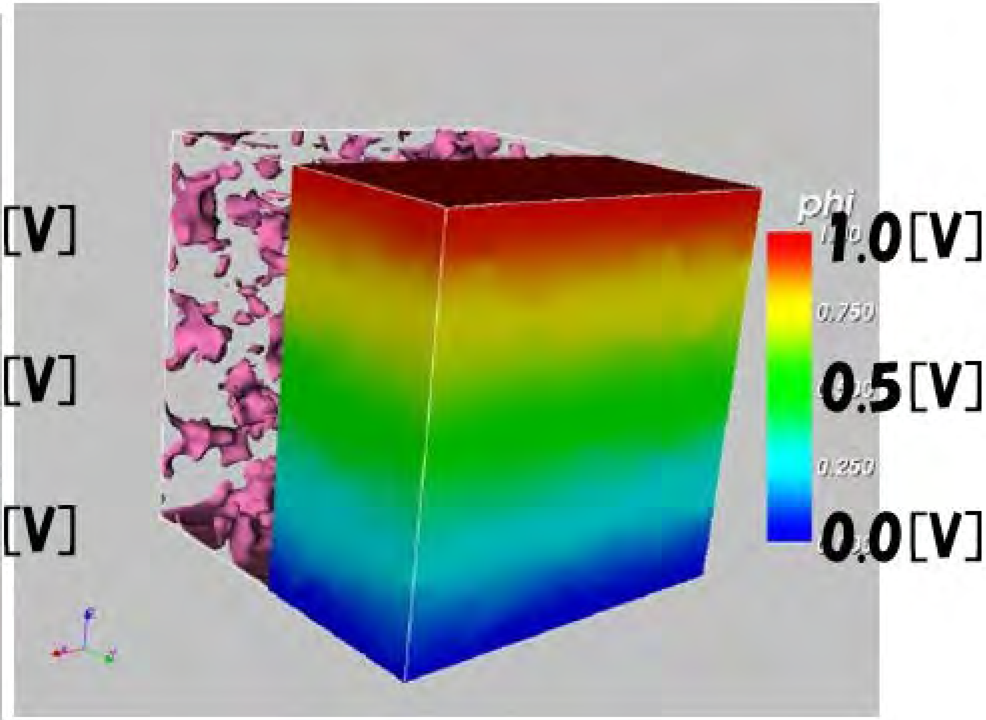
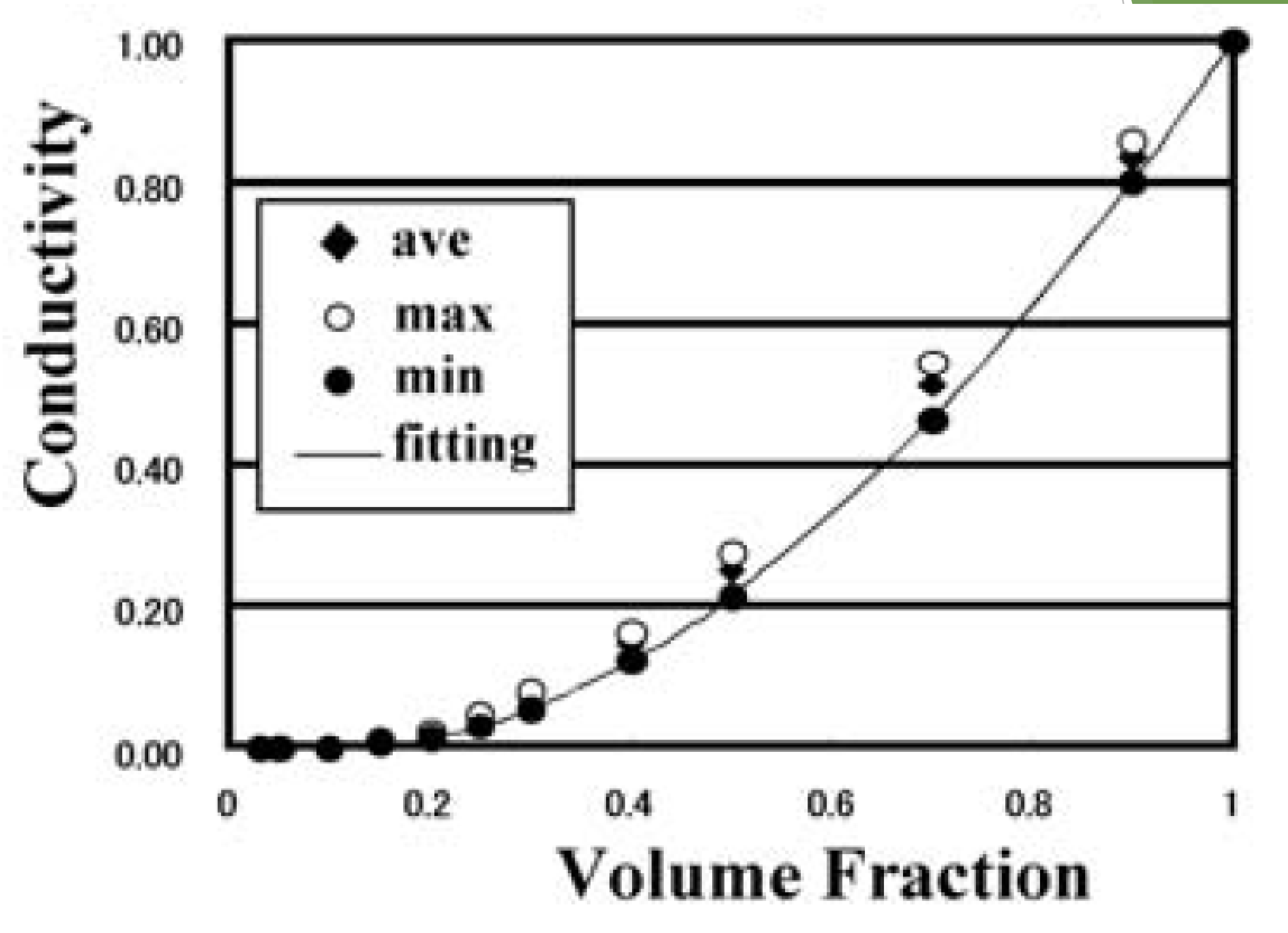


30%



90%

M-Shimosako-Wang, AMM 2013



1.00
0.80
0.60
0.40
0.20
0.00
Conductivity
ave
max
min
fitting
0
0.2
0.4
0.6
0.8
1
Volume Fraction

Picture of conductivity in randomized material.

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▶ ***We assume that the conductivity of AFCs obeys this picture.***

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This picture might be applicable to the design of a material, e.g., high resistance of sparse nano particle system of Pt, Au/Al_2O_3 (特許第4027373号, 第4448109号).

Advanced Mathematical Investigation for conductivity of highly disordered carbon systems; percolation and graph zeta function

1. Activation carbon fiber
2. Conductivity of ACFs
   Kuriyama’s Investigation
3. Conductivity of percolation
4. **Graph Theory**
5. New proposals on the conductivity
6. Summary

Graph theory

- Random Matrix theory
- Graph
- Graph zeta function
- Zeta function & RMT

Graph theory

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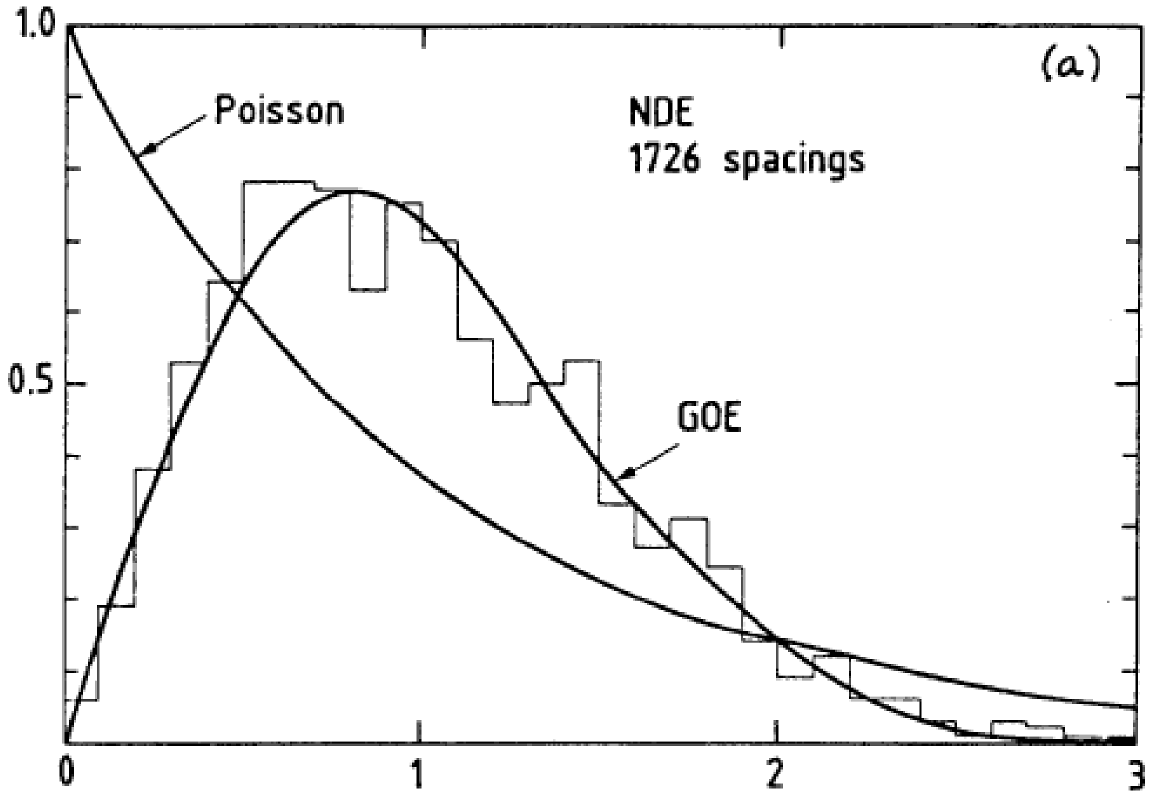
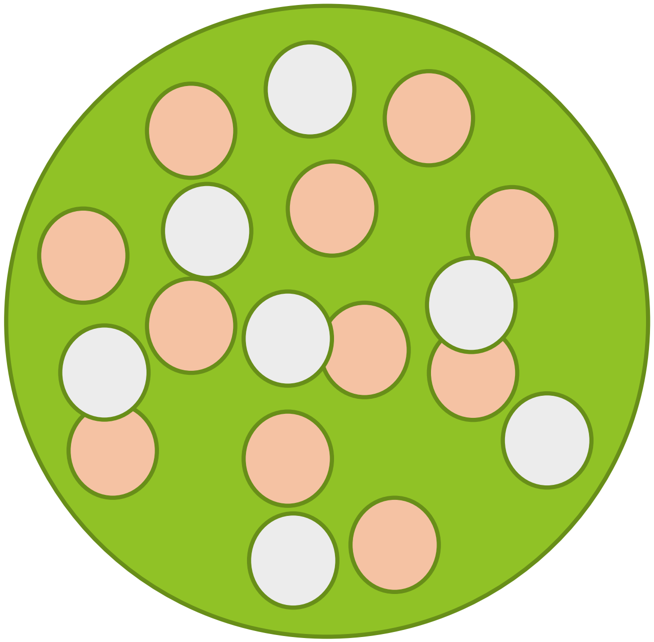
Random matrix theory appeared in the study of the analysis of scattering data for heavy nucleus.



Hamiltonian of many nucleons cannot be solved by perturbation theory because of strong forces and equivalence of each nucleon.

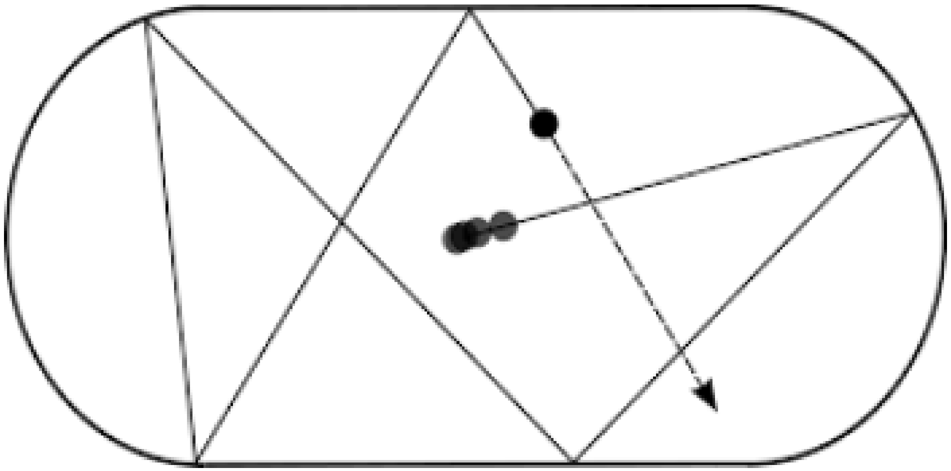


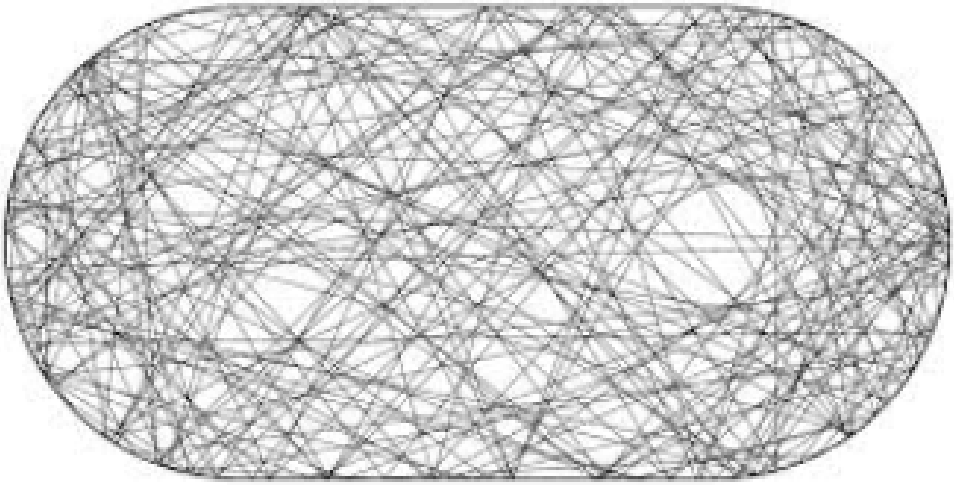
Hamiltonian of many nucleons cannot be solved by perturbation theory because of strong forces and equivalence of each nucleon.

Eigenvalue of Hamiltonian are complicated, i.e., the random matrix theory.

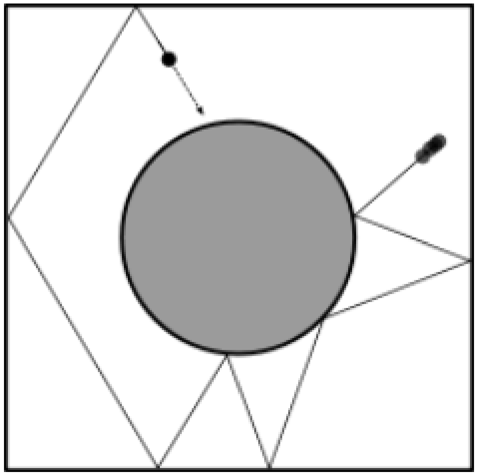
Quantum Chaos

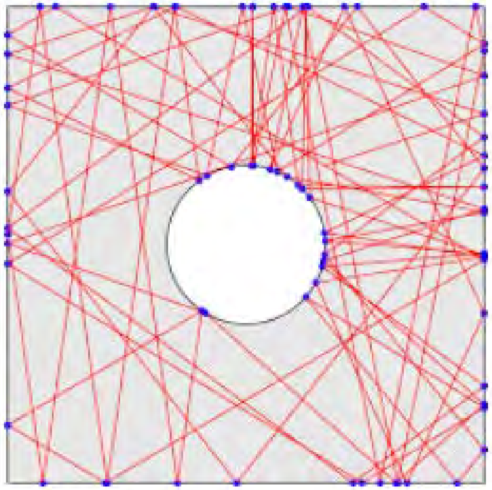
Classical billiard problems





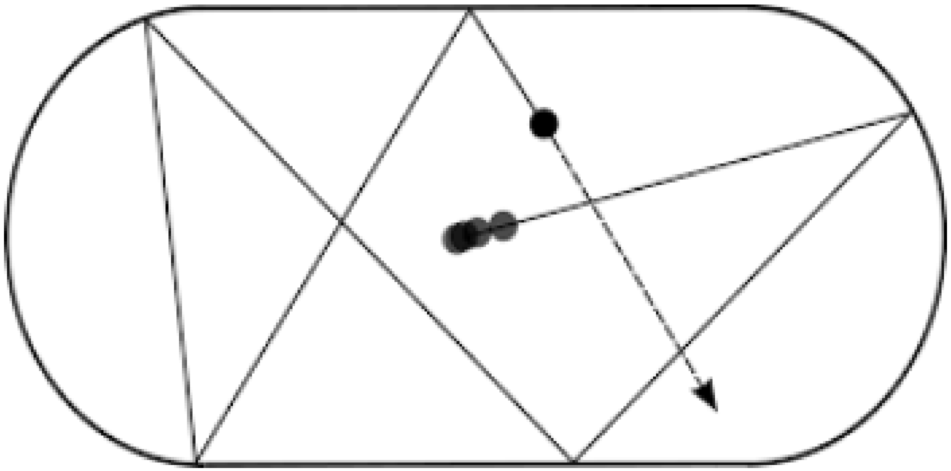
Sinai billiard

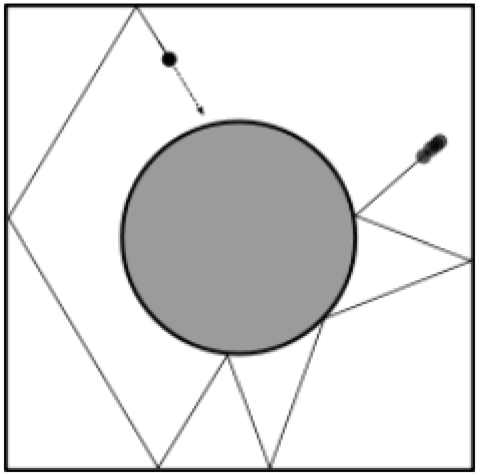


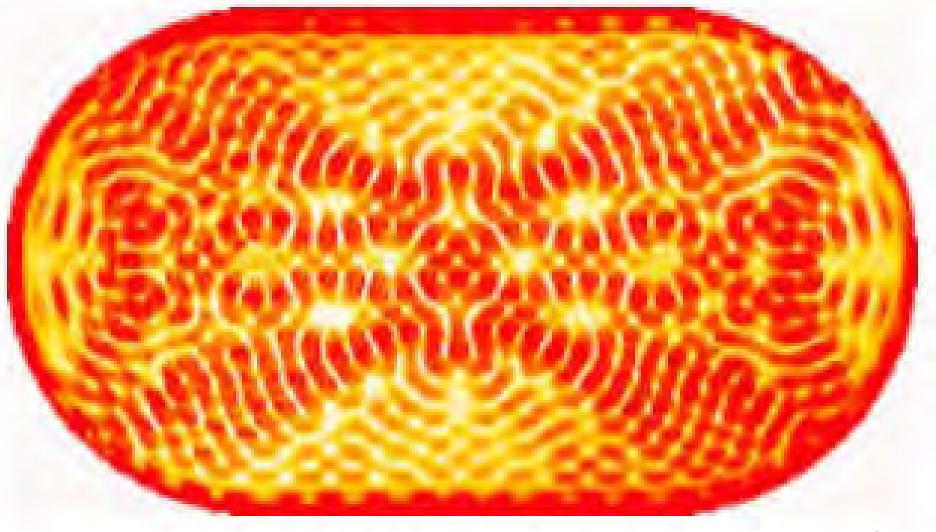


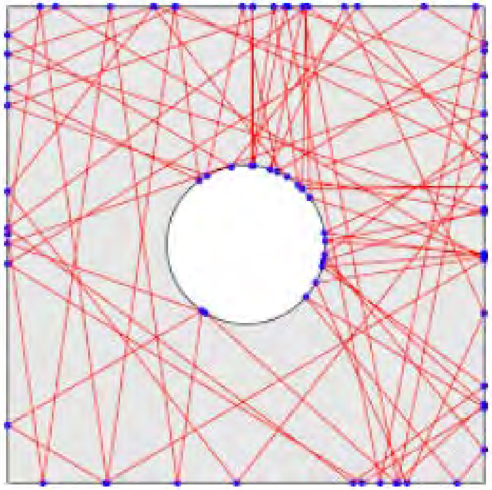
Quantum Chaos

Classical billiard problems ⇒ quantized!

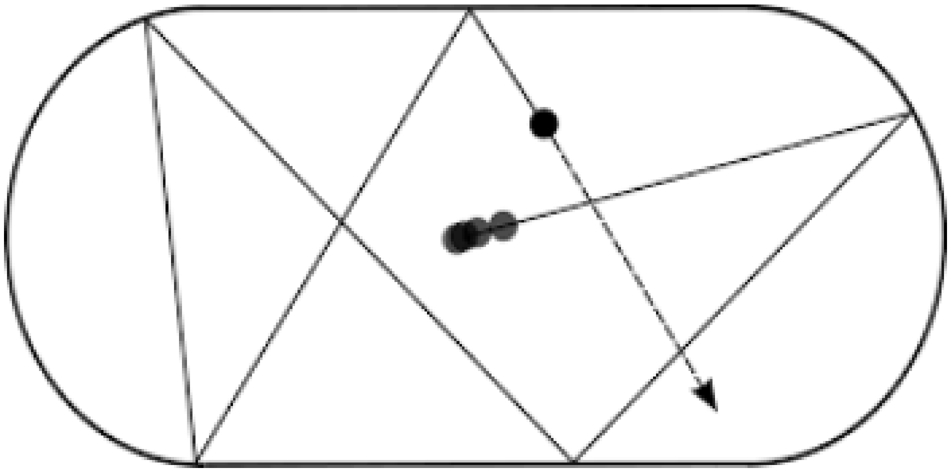


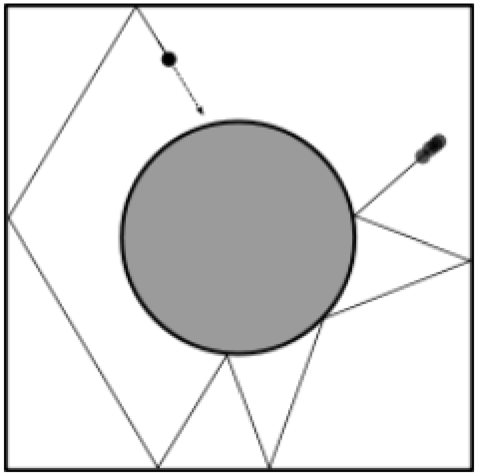


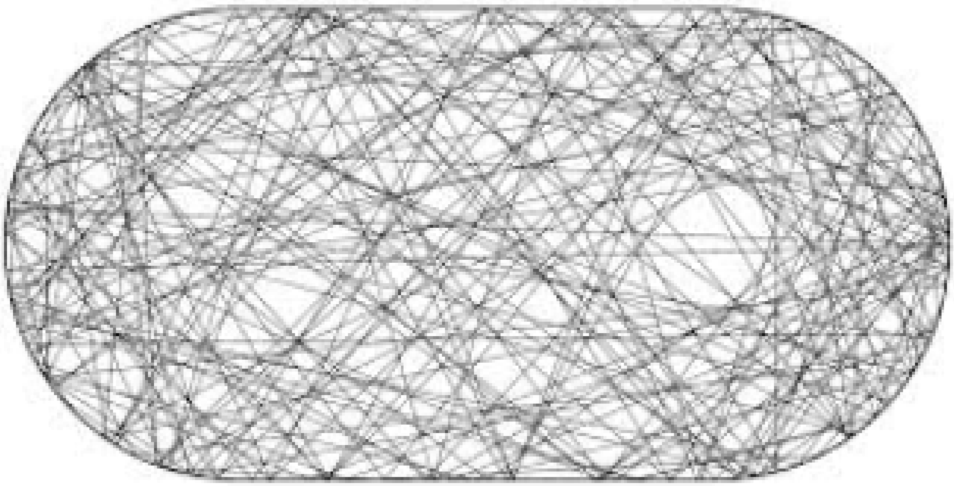


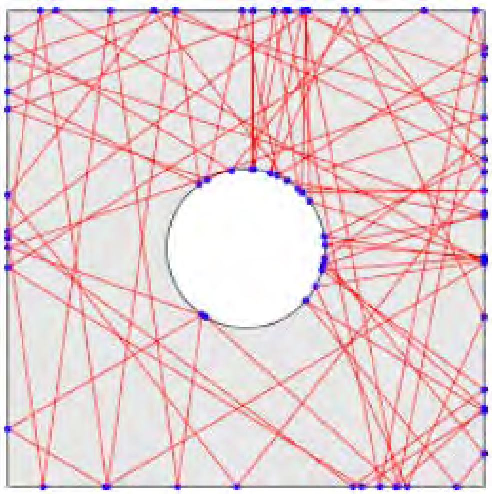


Quantum Chaos

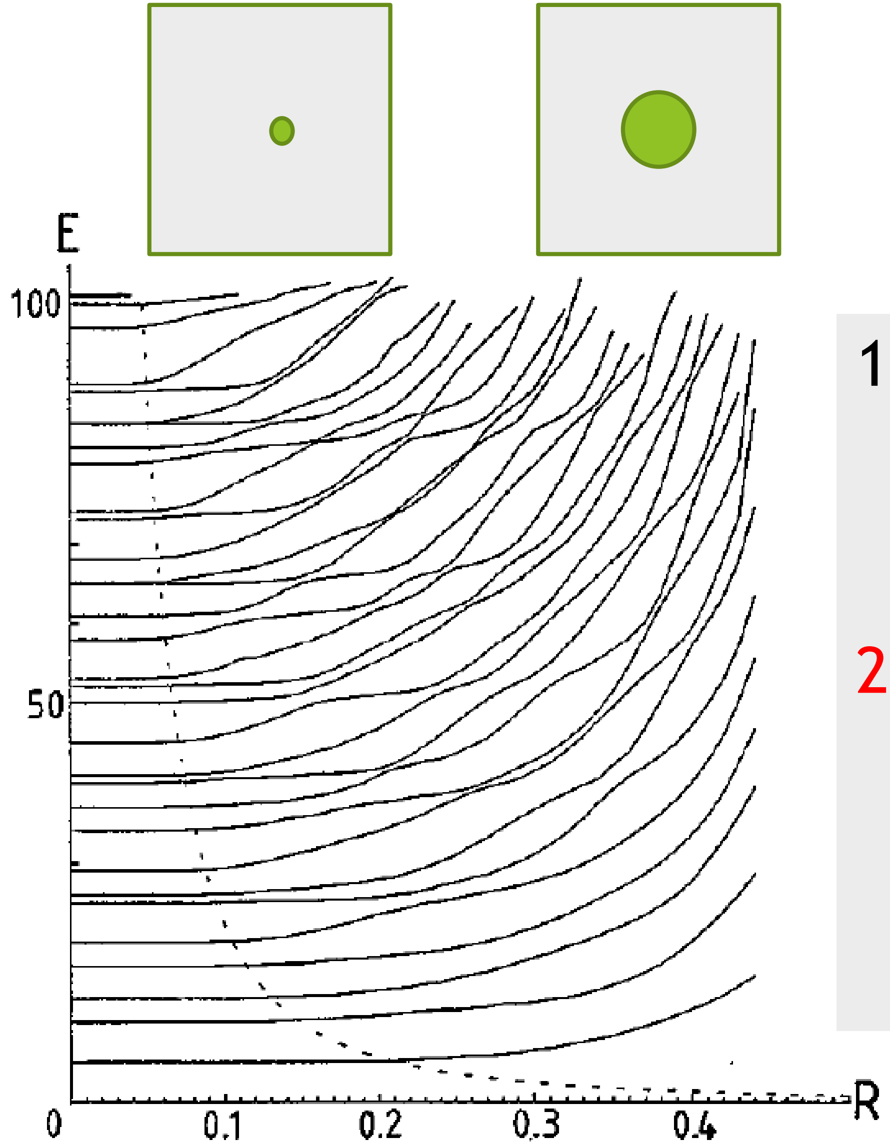








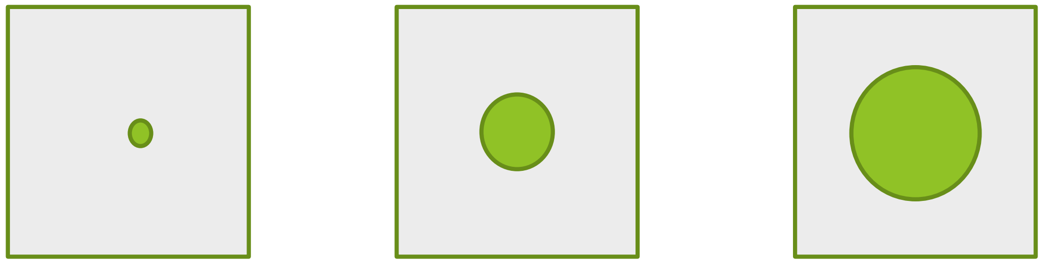
Quantum Chaos



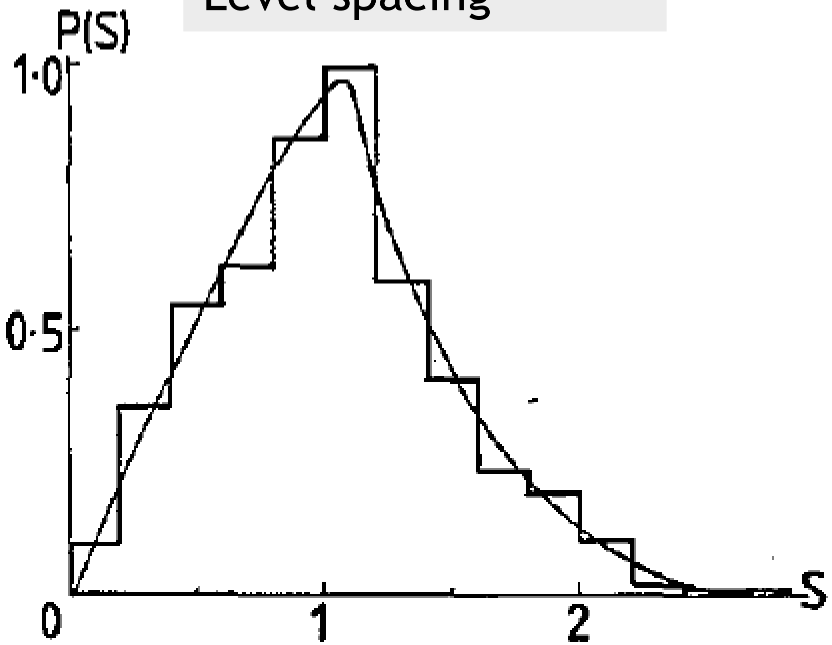
1. Eigenvalues of Hamiltonian cannot be expressed by perturbation theory
2. Each eigenvalues repels mutually because the states should not be degenerated due to the complexity.

M.Berry, 1981

Quantum Chaos



Level spacing



R for [0.2, 0.44]

1. Eigenvalues of Hamiltonian cannot be expressed by perturbation theory
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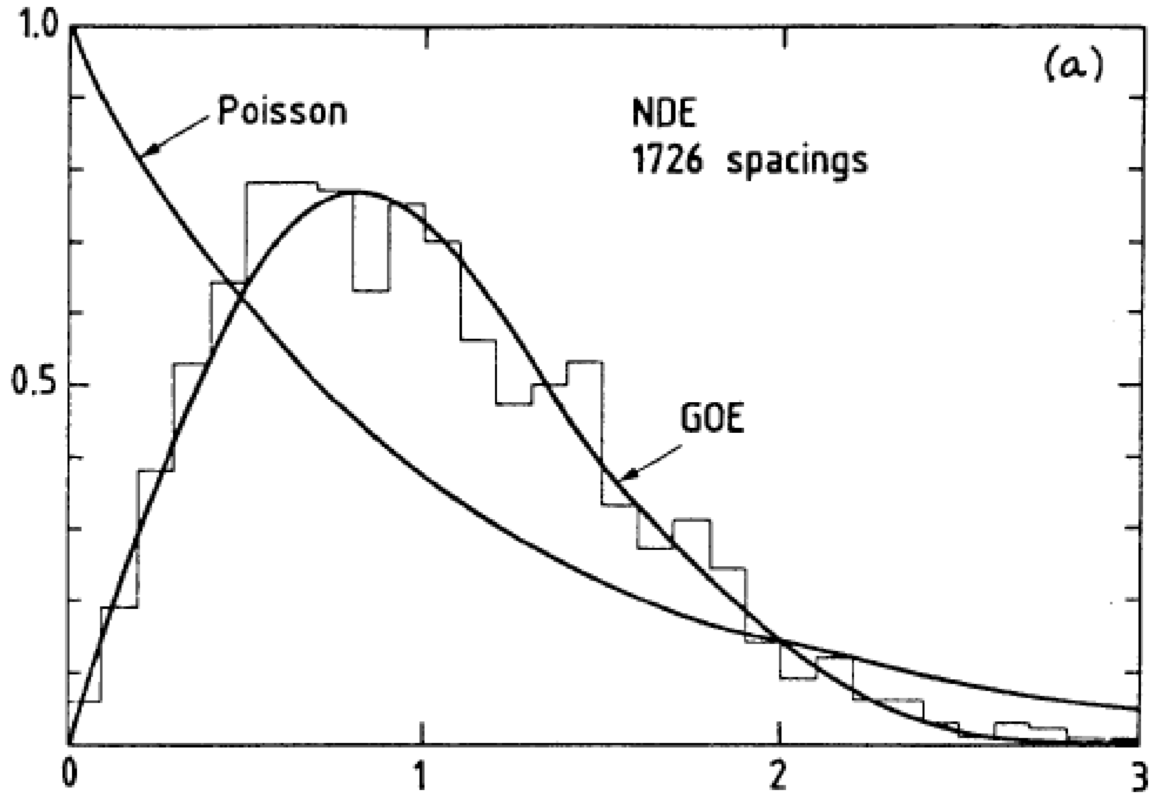
M.Berry, 1981

Random matrix theory for the study of the analysis of scattering data for heavy nucleus also governs the quantum chaos.



- Random Matrix theory

  $P = C \Pi dH_{ij} \exp(-a\, tr(H^2))$, under symmetry

  GOE (Gaussian Orthogonal Ensembles)

  GUE (Gaussian Unitary Ensembles)

  GSE (Gaussian Symplectic Ensembles)

	GOE	GUE	GSE
number	Real	Complex	Quotation
	Gaussian dist.	Gaussian dist.	Gaussian dist.
	$H=(A+A^T)/2$	$H=(A+A^*)/2$	$H=(A+A^D)/2$
symmetry	$OHO^T=H$	$UHU^*=H$	$S^DHS=H$
type	Symmetric	Hermite	Self-adjoint

- The eigenvalues of RM roughly obeys the Wigner semicircle,

$$\rho(t)dt \propto (\sigma^2 - t^2)dt$$

- The spacing of the eigenvalues of RM roughly obeys the Wigner surmise,

$$p(t)dt \propto t^{\beta} e^{-\alpha t^2} dt,$$

$\beta=1$, GOE,

$\beta=2$, GUE

	GOE	GUE	GSE
number	Real	Complex	Quotation
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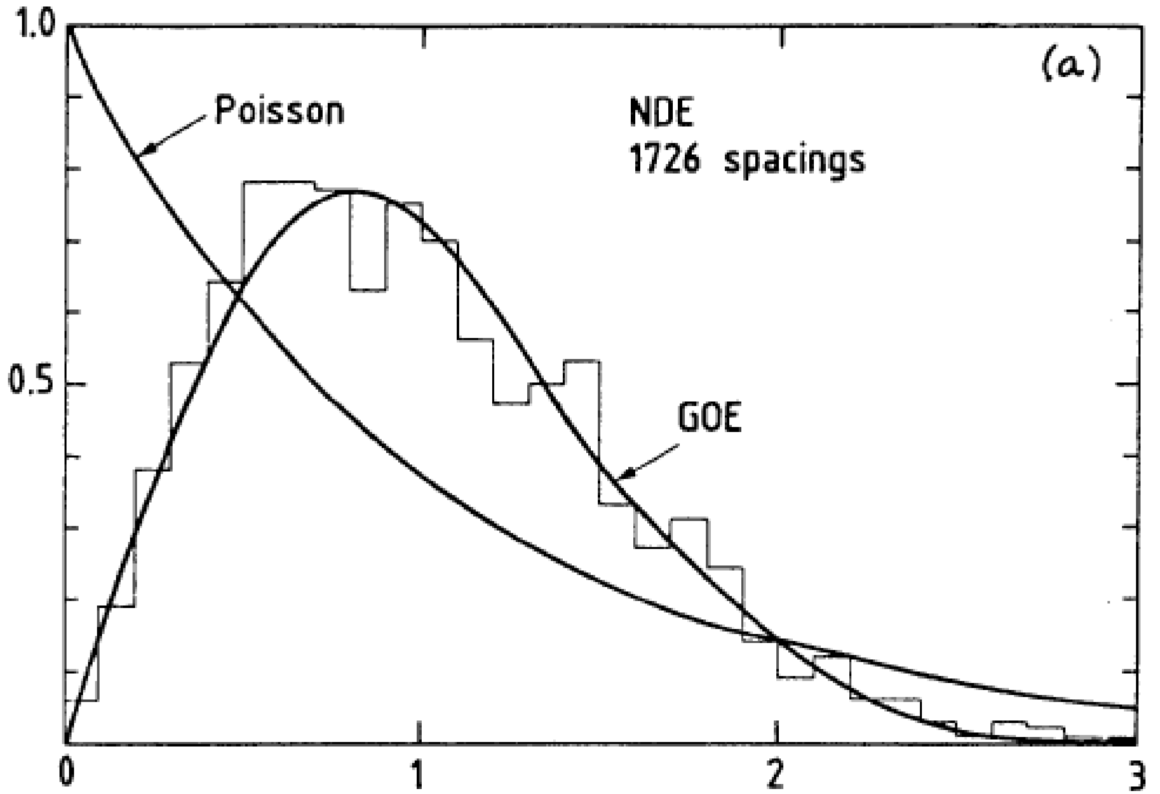
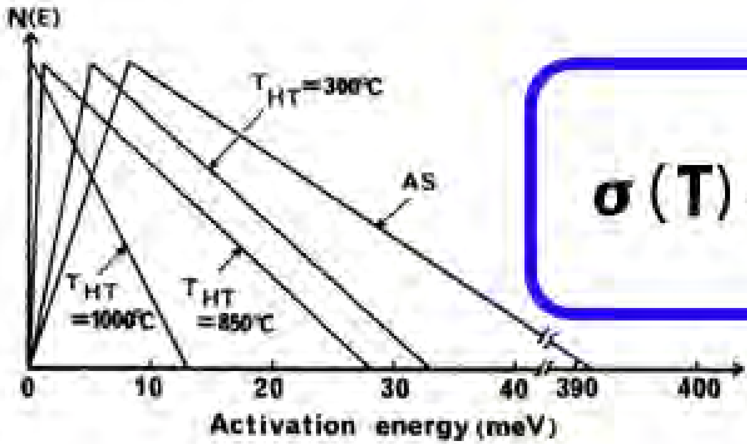


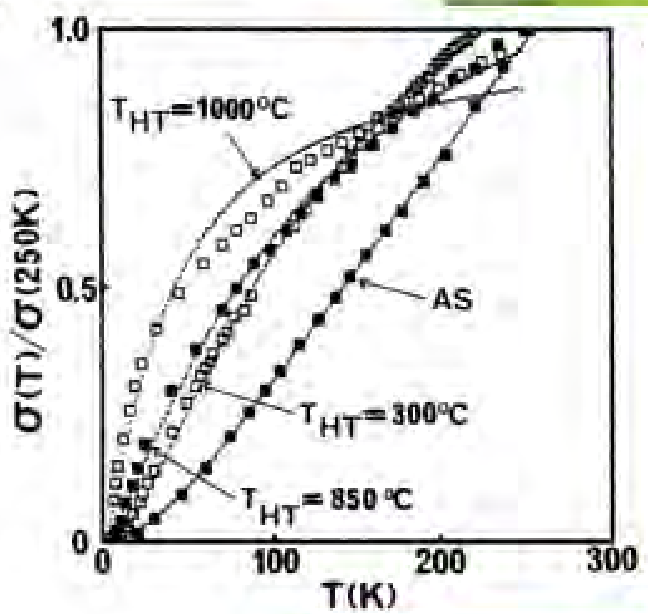
The Wigner surmise reminds us of the Kuriyama's density of state!



$$\sigma(T) = \sigma_0 \int N(E) e^{-E/T} dE$$

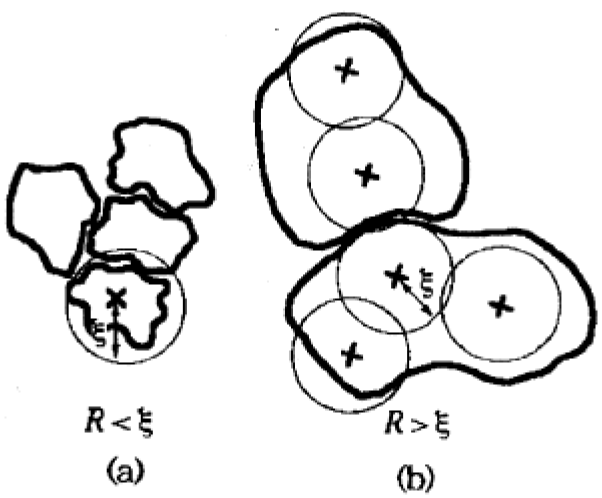
Kuriyama 's proposal recovers his experimental results

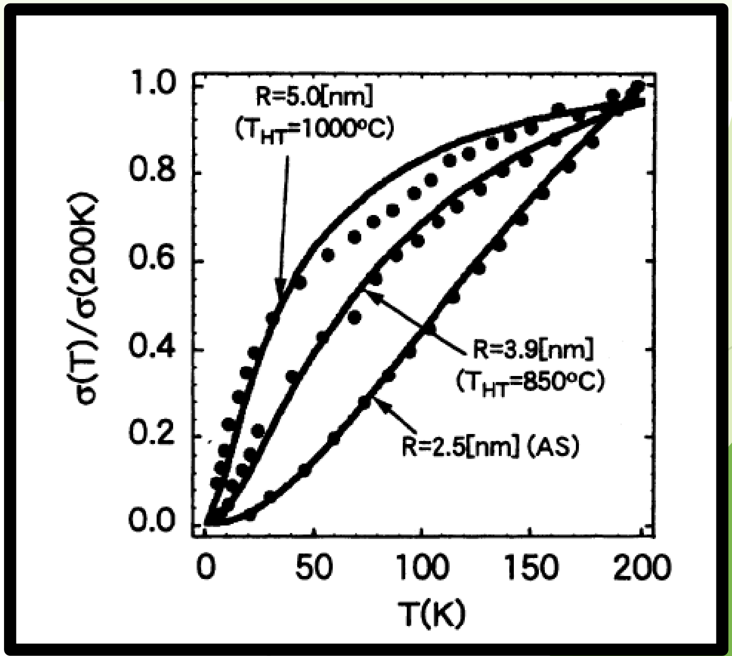
Phys.Rev.B 1993



Kubo phenomenon:
A property of a system with small particles might be determined by level spacing of small particles. (Though Kubo mistook a little bit)

Using it, M and A. Suzuki interpreted Kuriyama's phenomenological formula.
(PLA1996, PRB 2000)





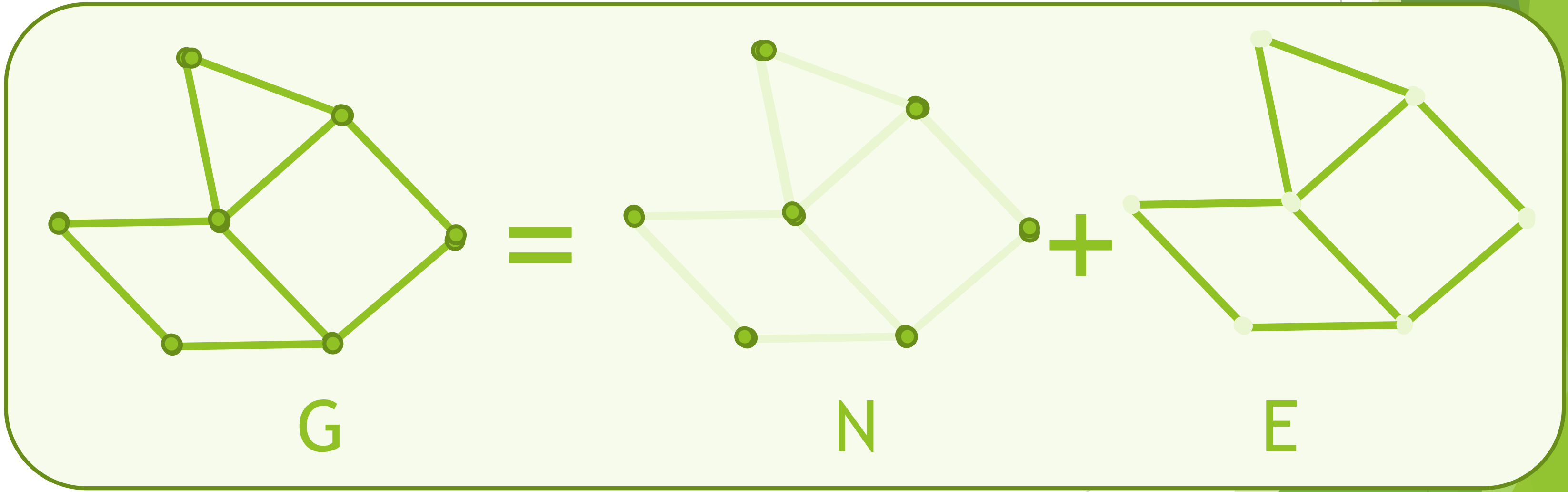
M. Suzuki PLA 1996

Graph theory

- Random Matrix theory
- Graph
- Graph zeta function
- Zeta function & RMT

Graph theory

G: graph consists of
a set of nodes N and a set of edges E



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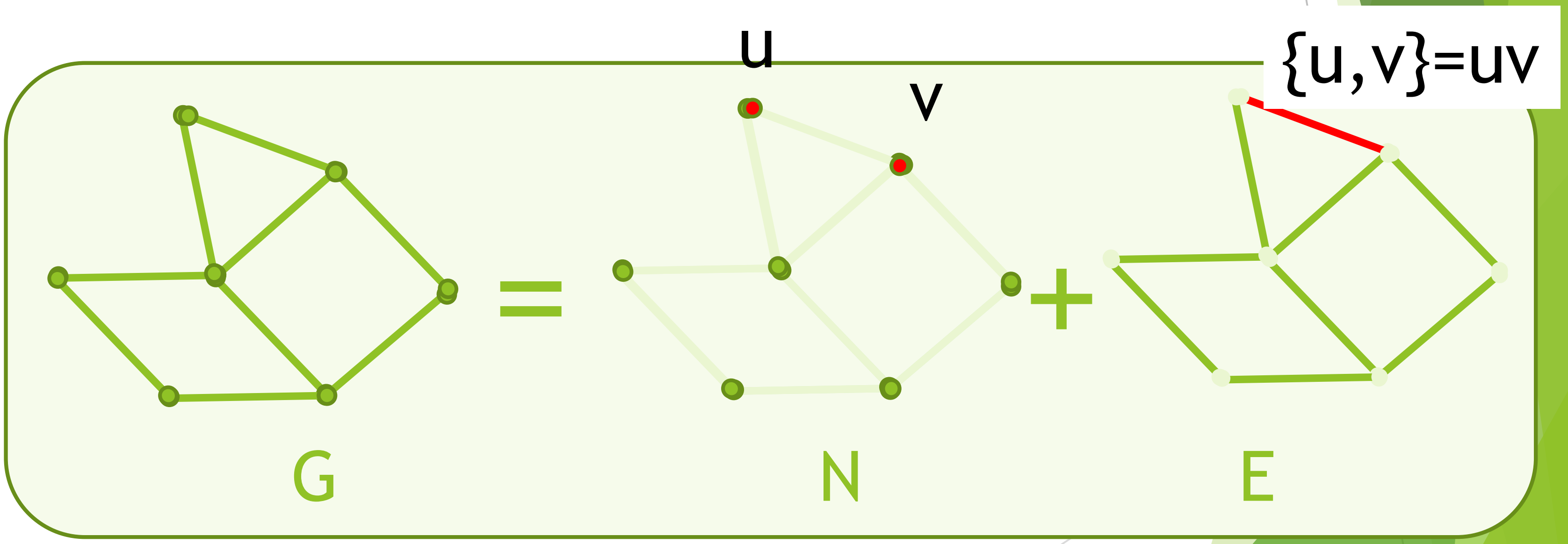
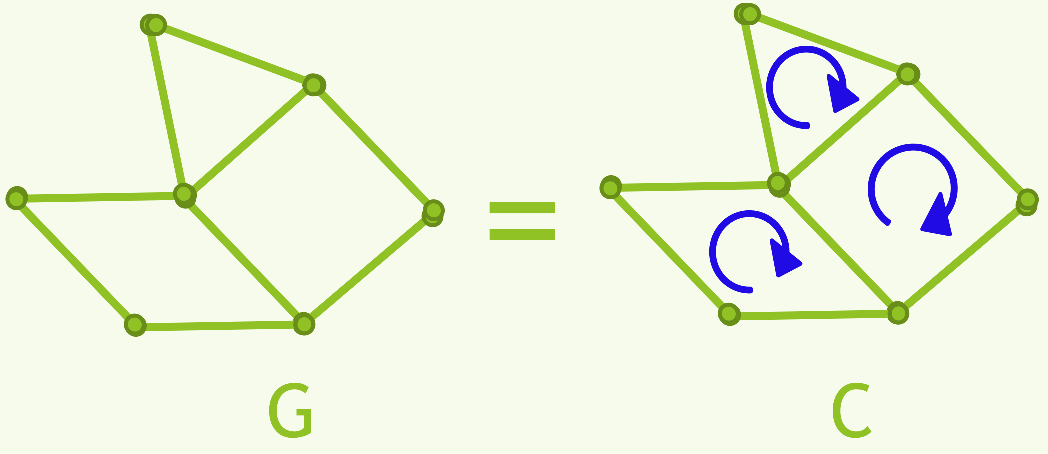


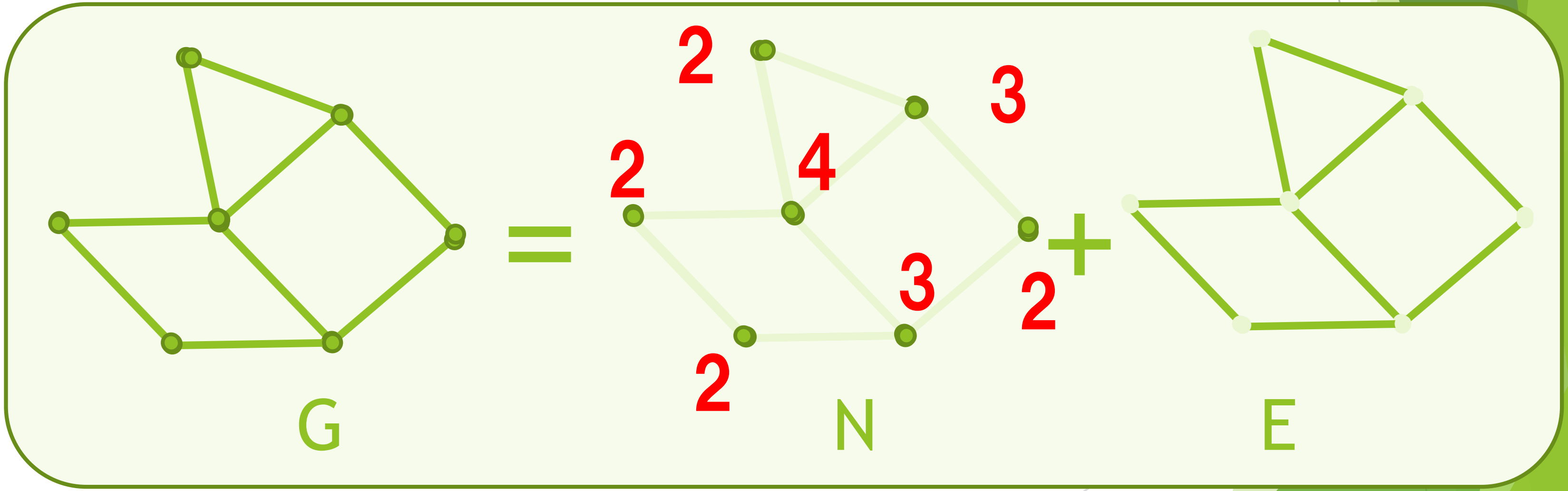
Graph theory

G: graph consists of
  a set of nodes N and a set of edges E
C: cycle of G



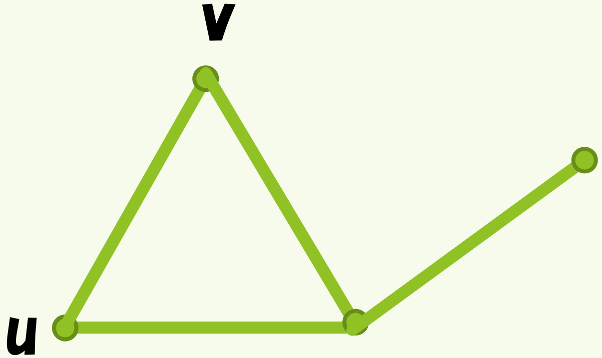
Graph theory

Degree of each node =
number of edges connected with it

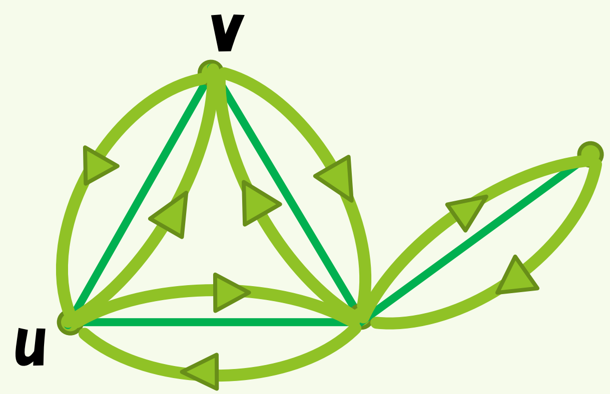


Ihara zeta functions

G: Finite simple connected graph



D_G: Symmetric digraph associated with G



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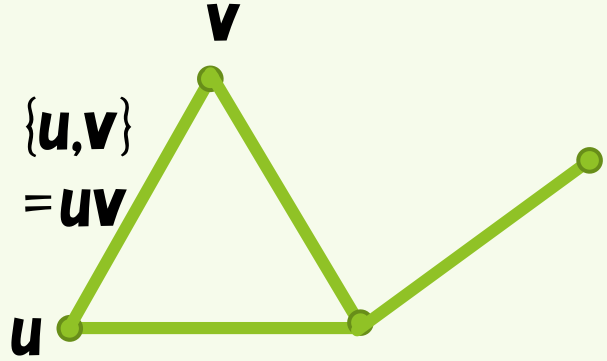

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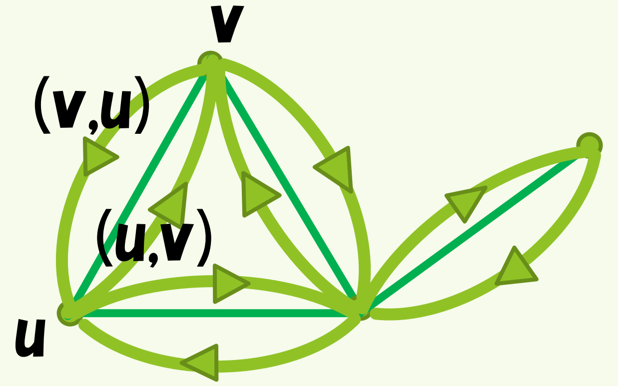

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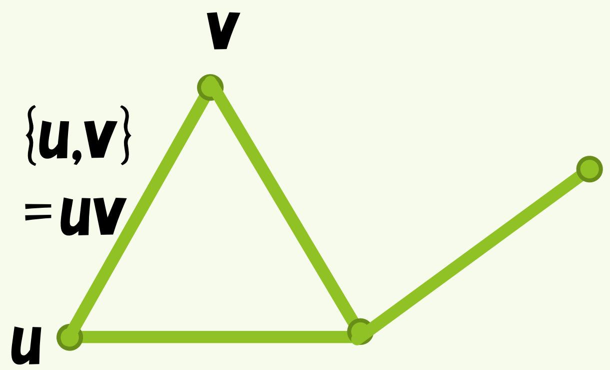

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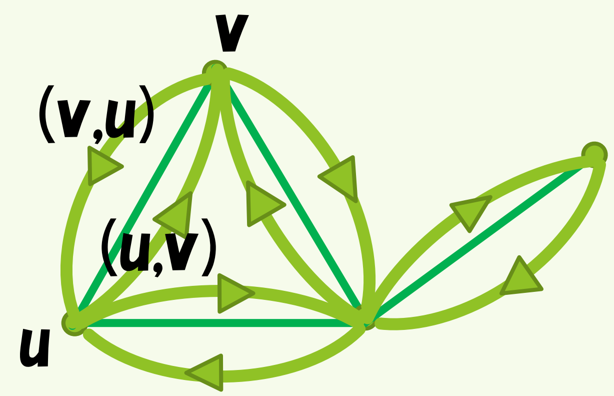
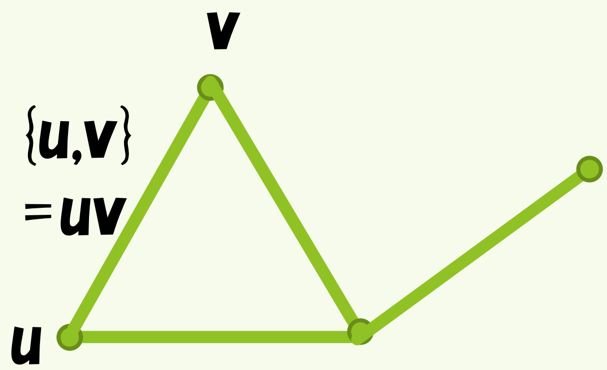


$D_G := \{ (u,v), (v,u) \mid uv \in G \}$

Ihara zeta functions

G: Finite simple connected graph



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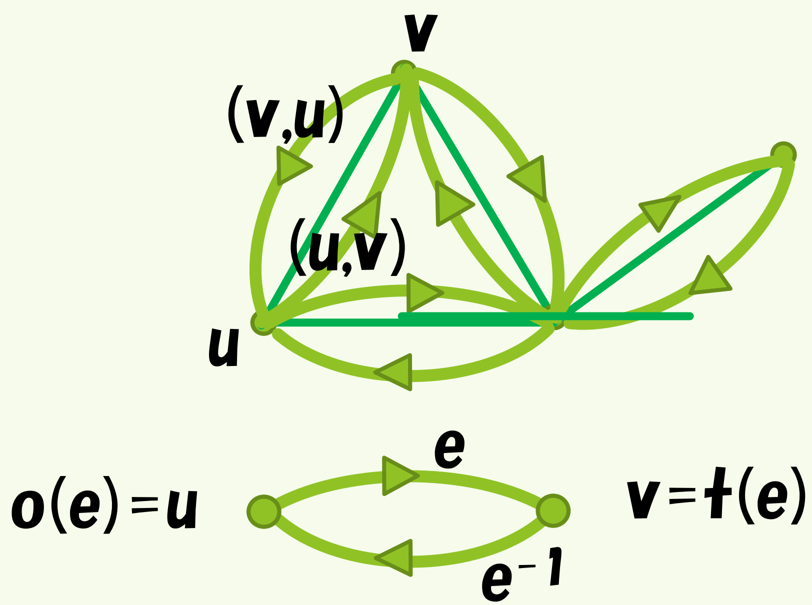
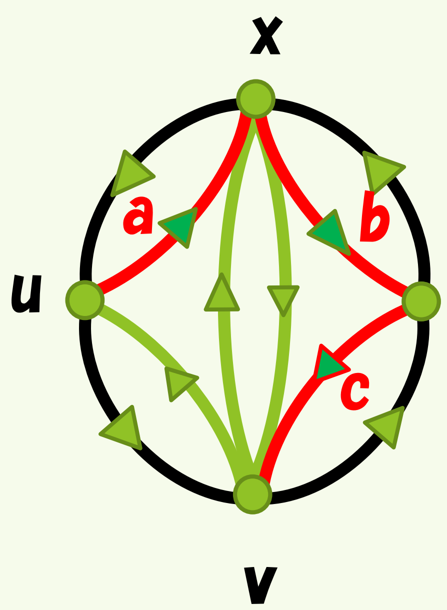


D_G: Symmetric digraph



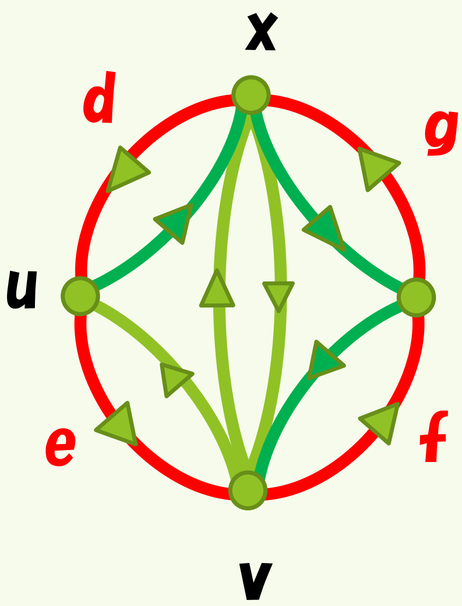
P: path

$P = (a,b,c)$

P: (u,v)-path

$|P|$: Length of P

$|P| = 3$

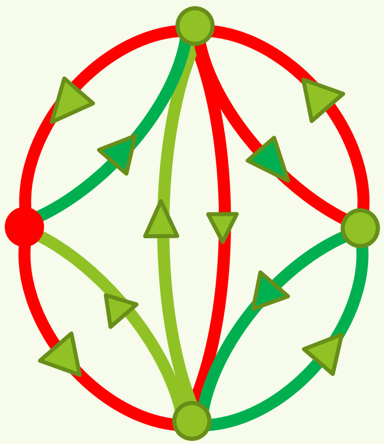


C: cycle

$C := (d, e, f, g)$

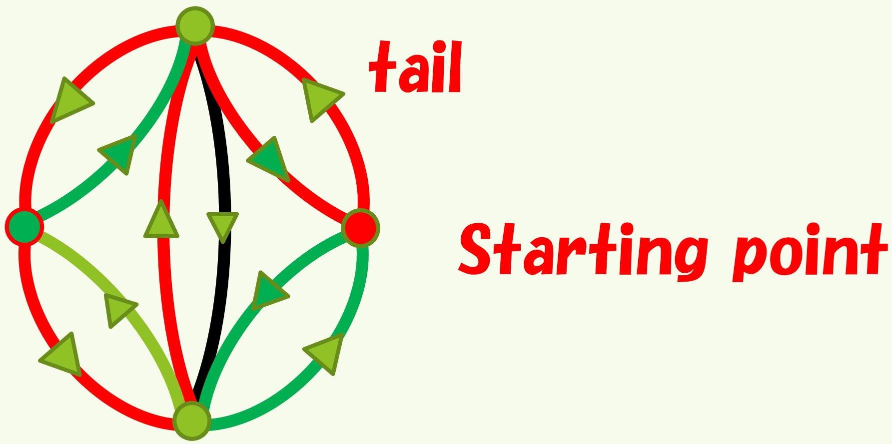
- (d,e,f,g),(e,f,g,d),(f,g,d,e),(dg,d,e,f) are equivalent.

[C]: equivalent class including C

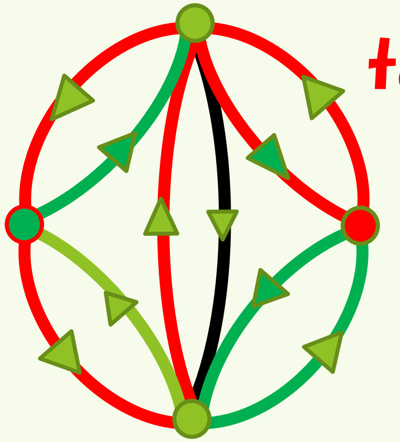


Backtracking
(bump)

Starting point



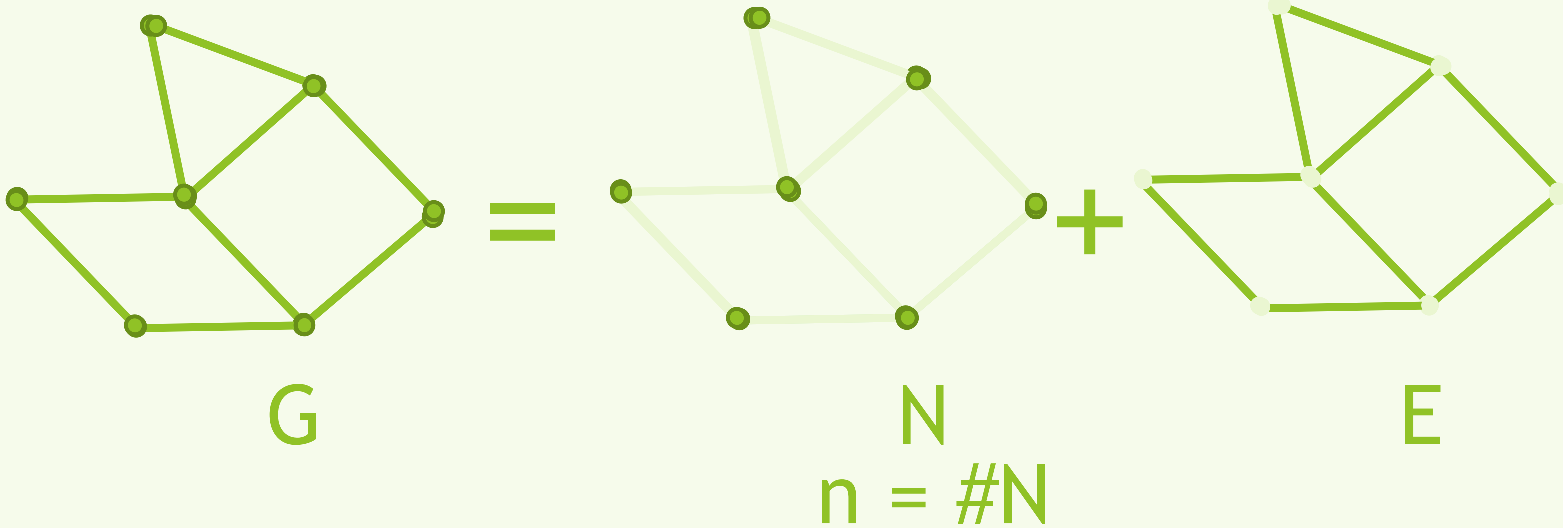
Cycle C: reduced ⇔
C does not contain backtracking & tail.



Starting point

Cycle C: prime ⇔
C does not equal to B^r for a certain cycle B

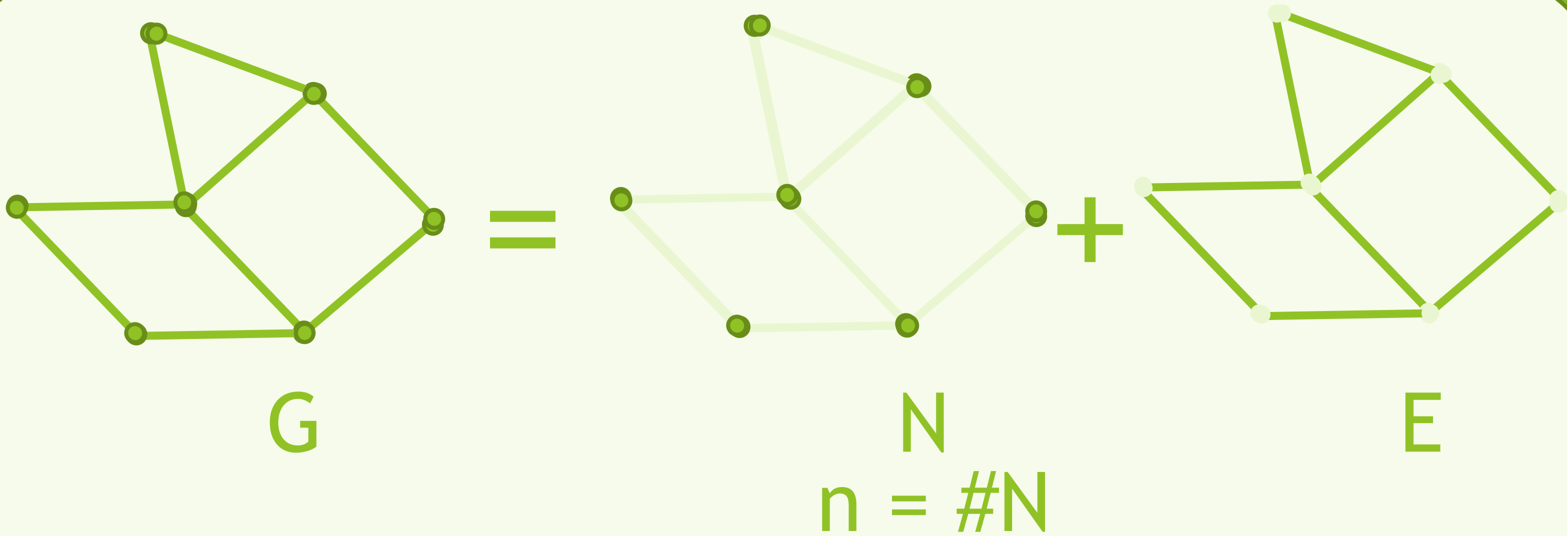
Graph theory



Adjacency matrix: n x n matrix

$$A_G = (a_{ij}) = \begin{cases} 1 & \text{If } (i,j) \in E \\ 0 & \text{otherwise} \end{cases}$$

Graph theory



Degree matrix: n x n matrix

$$D_G = (d_{ij}) = \begin{cases} \text{degree of } i & \text{if } i=j \\ 0 & \text{otherwise} \end{cases}$$

Graph G: Ihara zeta function,

$$Z(G,u) = \prod_{[C]} (1 - u^{|C|})^{-1}$$

[C]:equivalent class of prime reduced cycles of G

Graph theory

- Random Matrix theory
- Graph
- **Graph zeta function**
- Zeta function & RMT

Graph G: Ihara zeta function,

$$Z(G,u) = \prod_{[C]} (1 - u^{|C|})^{-1}$$

[C]:equivalent class of prime reduced cycles of G

Graph G: Ihara zeta function,

$$Z(G,u) = \prod_{[C]} (1 - u^{|C|})^{-1}$$

$$= \exp\left(\sum_{k=1}^{\infty} \frac{N_k}{k} u^k \right)$$

N_k:number of reduced k-cycles.

Graph G: Ihara zeta function,

$$Z(G,u) = \prod_{[C]} (1 - u^{|C|})^{-1}$$

$$= (1-u^2)^{n-m} \det(I - uA_G + u^2 (D_G - I))^{-1}$$

m=#E, n=#N,

I : nxn unit matrix

A_G: adjacency matrix

D_G : degree matrix

Graph G: Ihara zeta function,

$$Z(G,u) = \prod_{[C]} (1 - u^{|C|})^{-1}$$

$$= (1-u^2)^{n-m} \det(I - uA_G + u^2 (D_G - I))^{-1}$$

The Ihara zeta is related to the adjacency matrix.

(e.g., Kotani-Sunada 2000)

Graph theory

- Random Matrix theory
- Graph
- Graph zeta function
- Zeta function & RMT

Graph zeta function

G: graph consists of
a set of nodes N and a set of edges E
C: cycle of G

Ihara zeta function

$$Z(G,u) = \prod_{C} (1 - u^{|C|})^{-1}$$

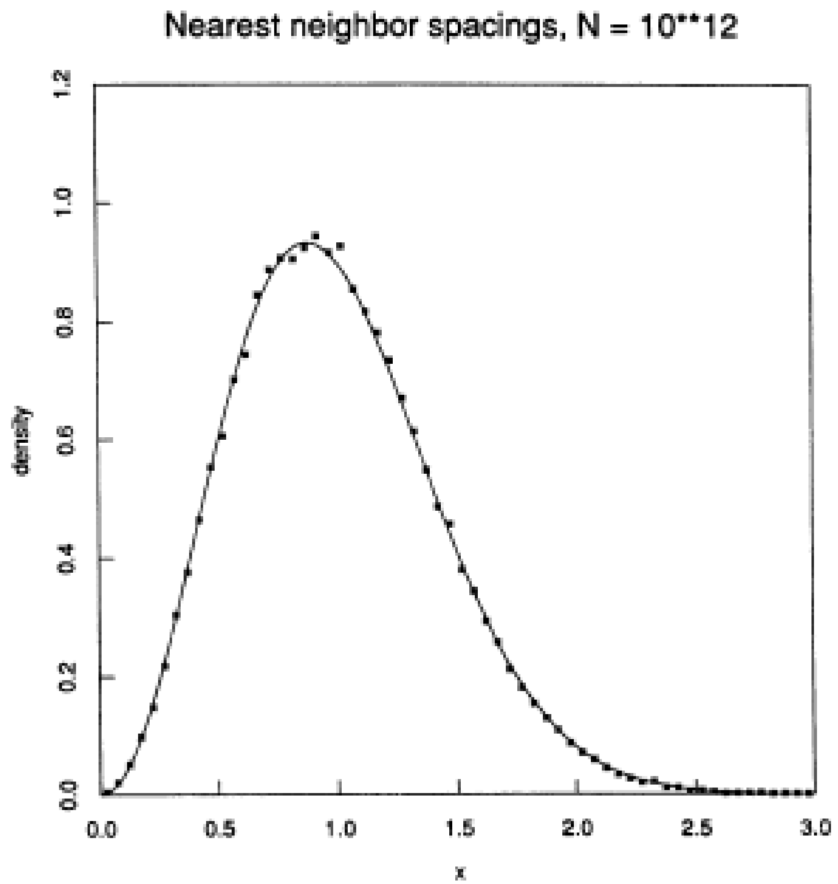
zeta function

- Riemann zeta function:

$$\zeta(s) = \sum_{n \in N} \frac{1}{n^s} = \prod_{p\,:\text{prime}} \frac{1}{(1-p^{-s})}$$

- Riemann hypothesis is
  "Real parts of non-trivial zeros of ζ
  belong to ½"
- Under RH, interval of the non-trivial zeros of ζ roughly obeys the Wigner surmise,

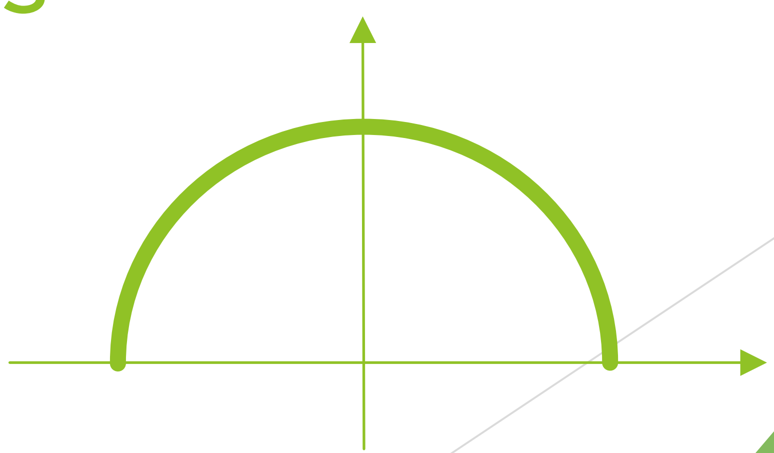
$$p(t)dt \propto t^{\beta} e^{-\alpha t^2} dt, \quad \beta=2$$



Odlyzko 1987

Graph zeta function and Random matrix theory

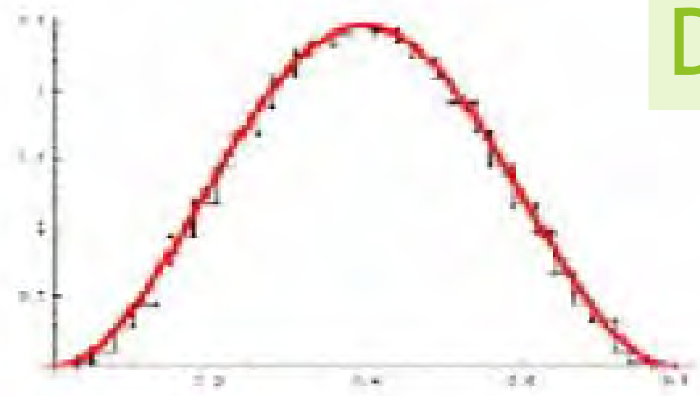
Sunada (2001)
For a certain family of graphs, the asymptotic behavior of the distributions of eigenvalues of adjacency matrices obeys the Wigner semi-circle law.



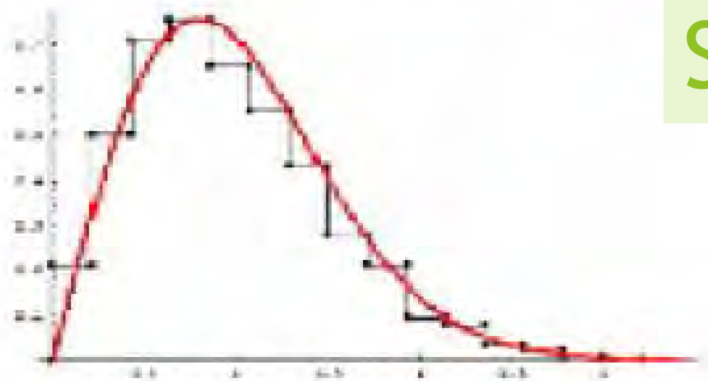
Newland (2005)

Audrey Terras
"Zeta Functions of Graphs" 2010

For a pseudo-random regular graph with degree 53 and 2000 vertices, the imaginary parts of poles of the Ihara zeta function.



Distribution



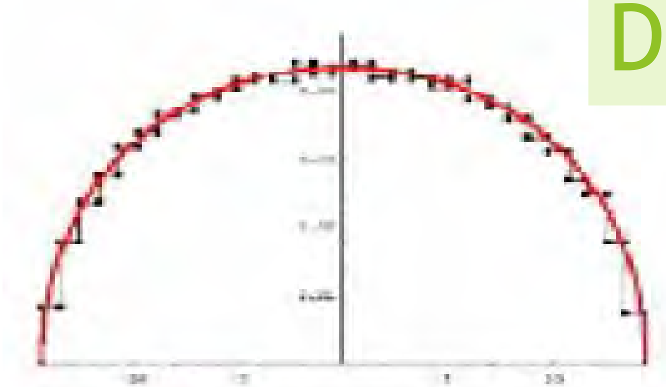
Spacing distribution

$$p(t)dt \propto te^{-\alpha t^2}dt$$

Newland (2005)

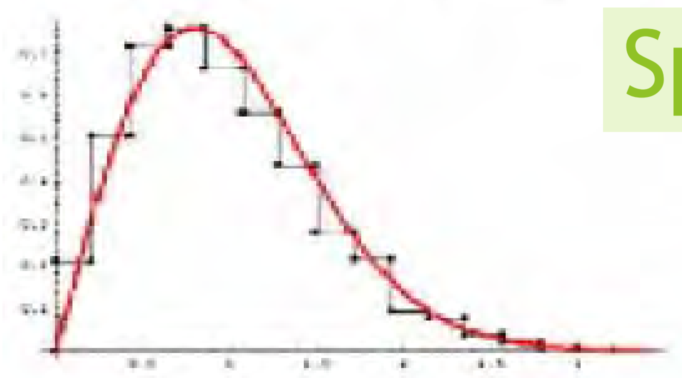
Audrey Terras
"Zeta Functions of Graphs" 2010

For a pseudo-random regular graph with degree 53 and 2000 vertices, the eigenvalues of the Adjacency matrix.



Distribution

$\rho(t)dt \propto (1-t^2)dt$



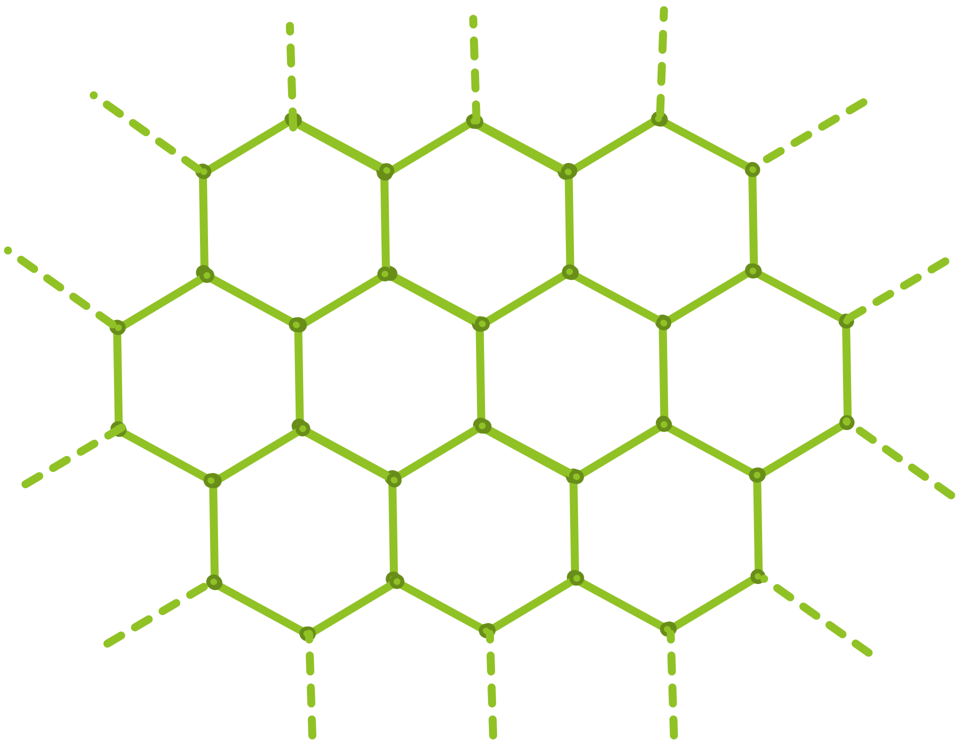
Spacing distribution

$p(t)dt \propto te^{-\alpha t2}dt$

Advanced Mathematical Investigation for conductivity of highly disordered carbon systems; percolation and graph zeta function

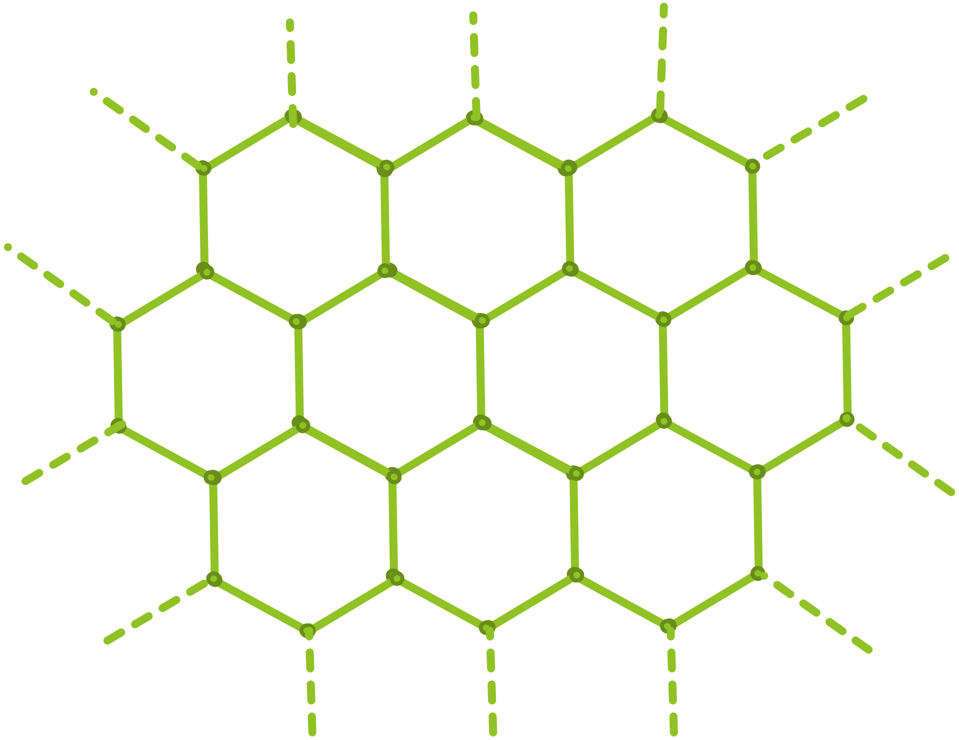
1. Activation carbon fiber
2. Conductivity of ACFs
   Kuriyama's Investigation
3. Conductivity of percolation
4. Graph Theory
5. **New proposals on the conductivity**
6. Summary

Apply RM and GZFT to carbon system.



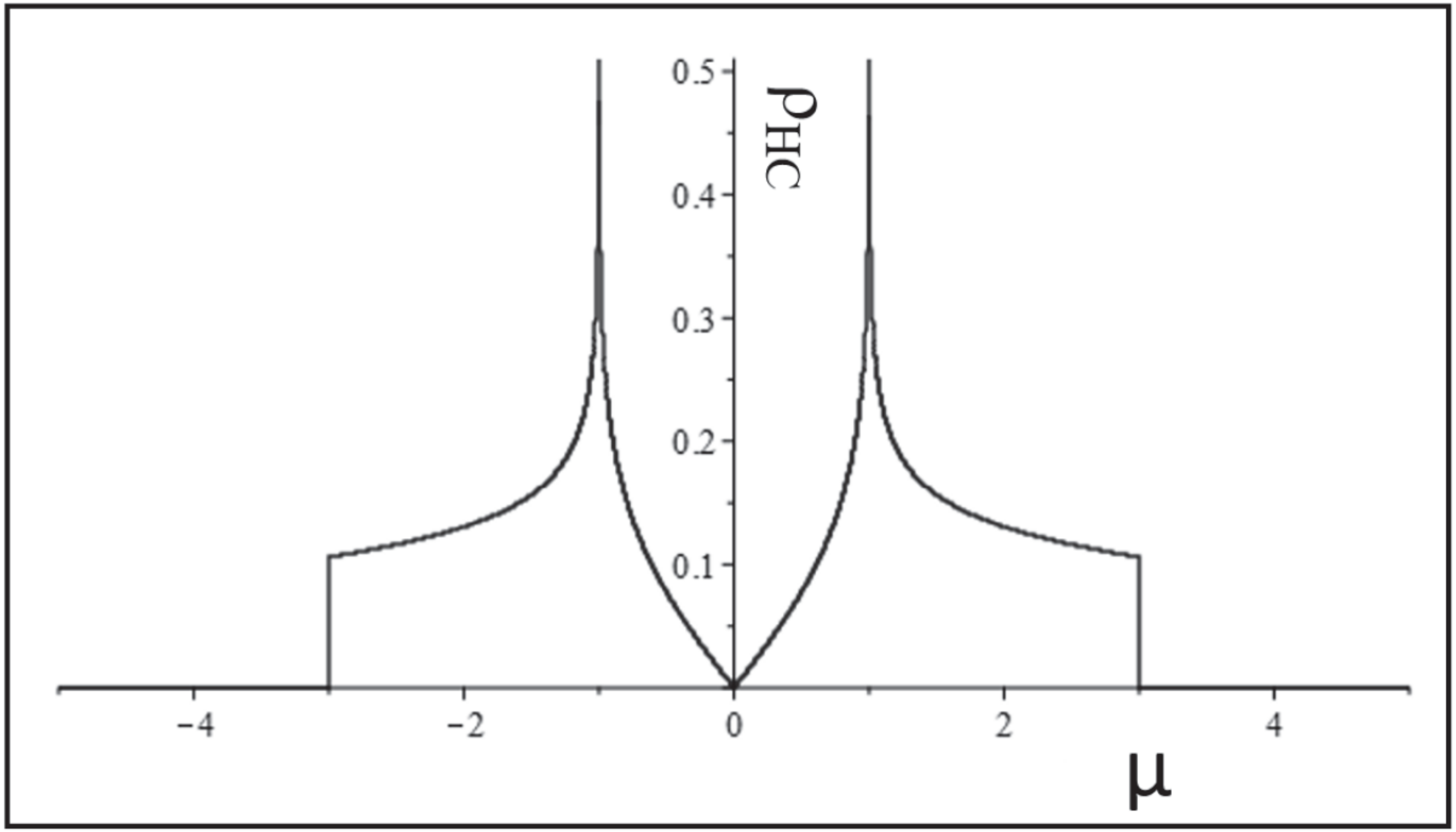
Eigenvalues of Adjacency matrix

For the case of Infinite Honeycomb lattice



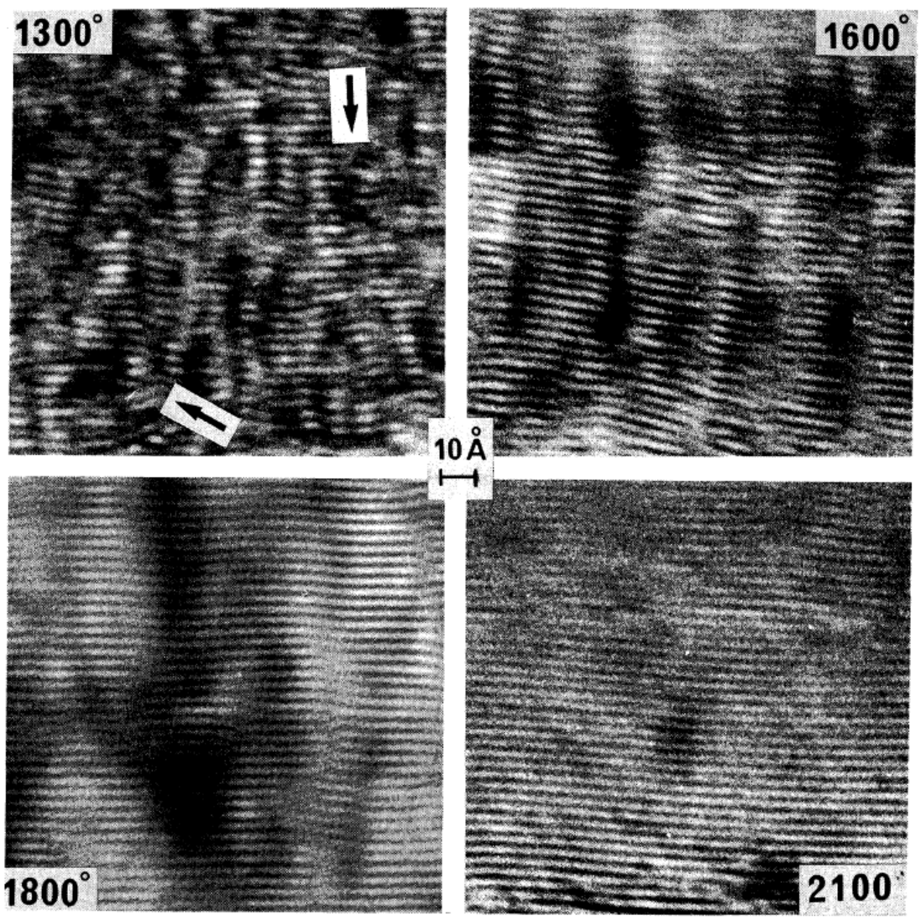
Eigenvalues of Adjacency matrix are give by the complete elliptic integrals.

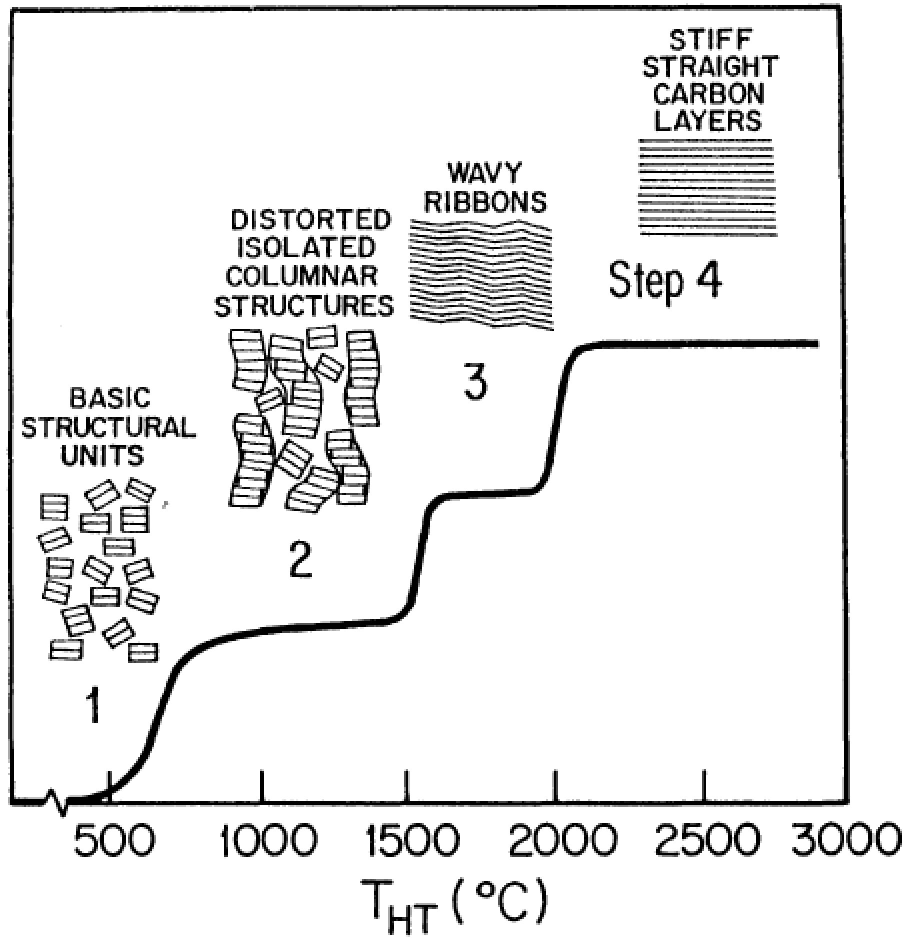
Distribution of the eigenvalues of the adjacency matrix



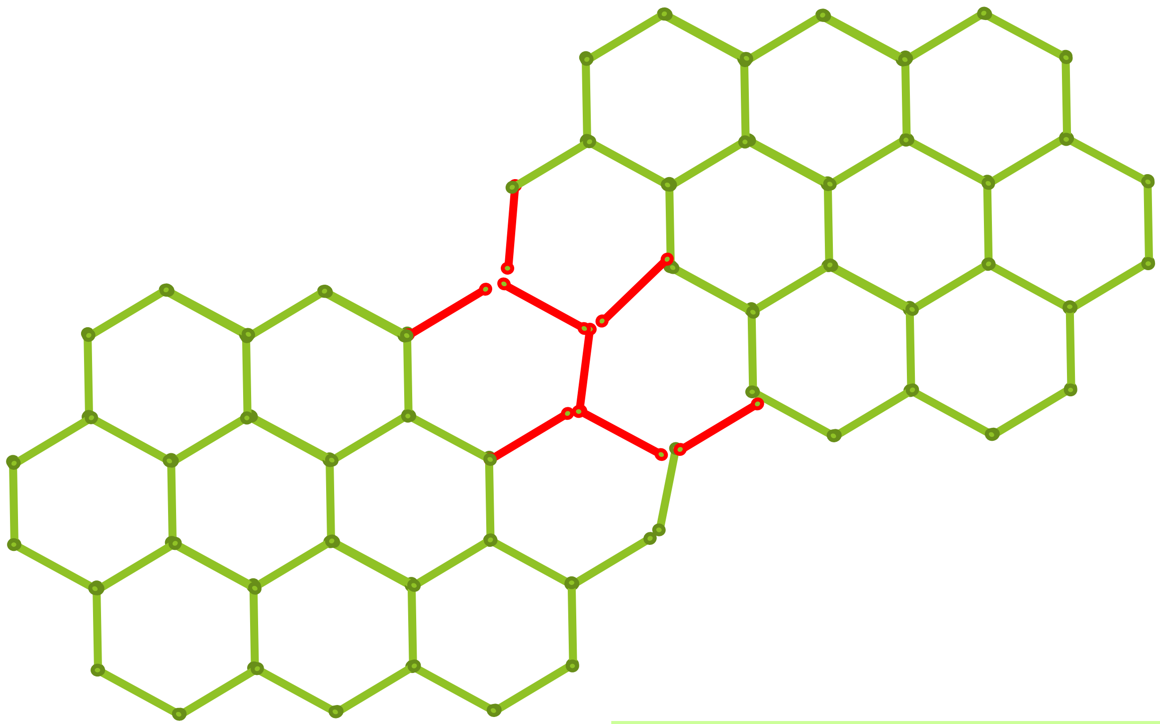
Using the lattice Green's function, the eigenvalues are given by the complete elliptic integrals. T. Horiguchi, J. Math. Phys. 19 (1972) 1411–1419.

Apply RM and GZFT to disordered carbon system.



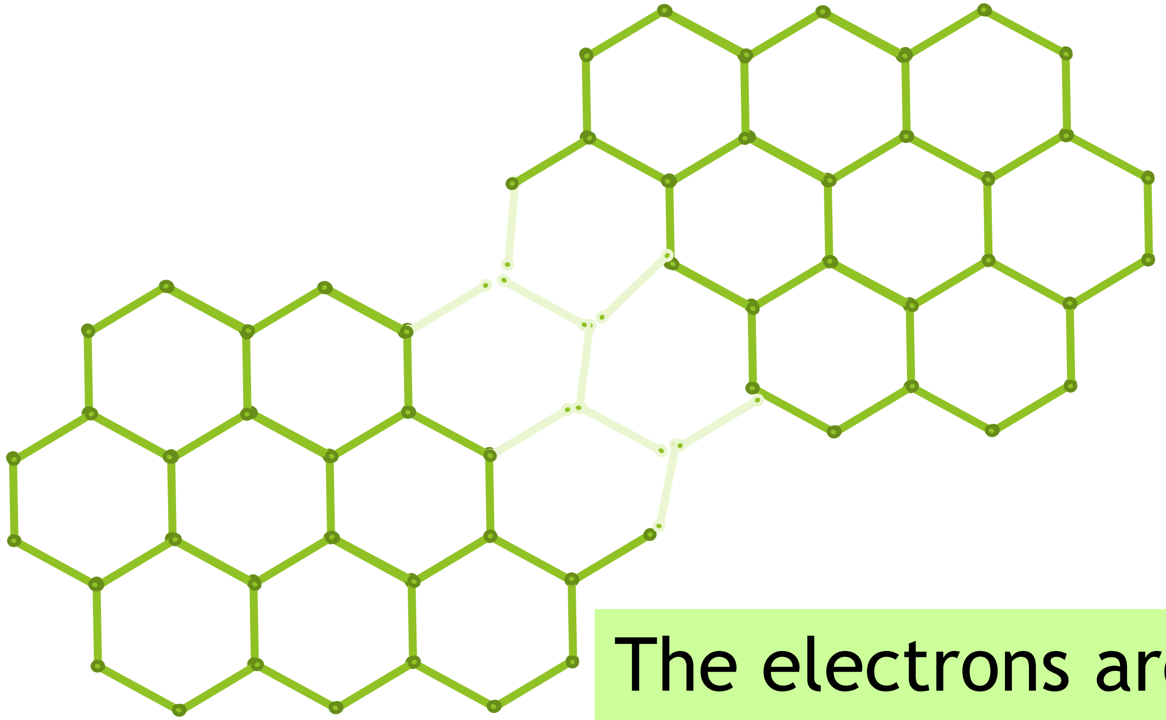


At the connection of the graphite pieces, the phonons are easily excited and coherency of the electron wave function is lost there



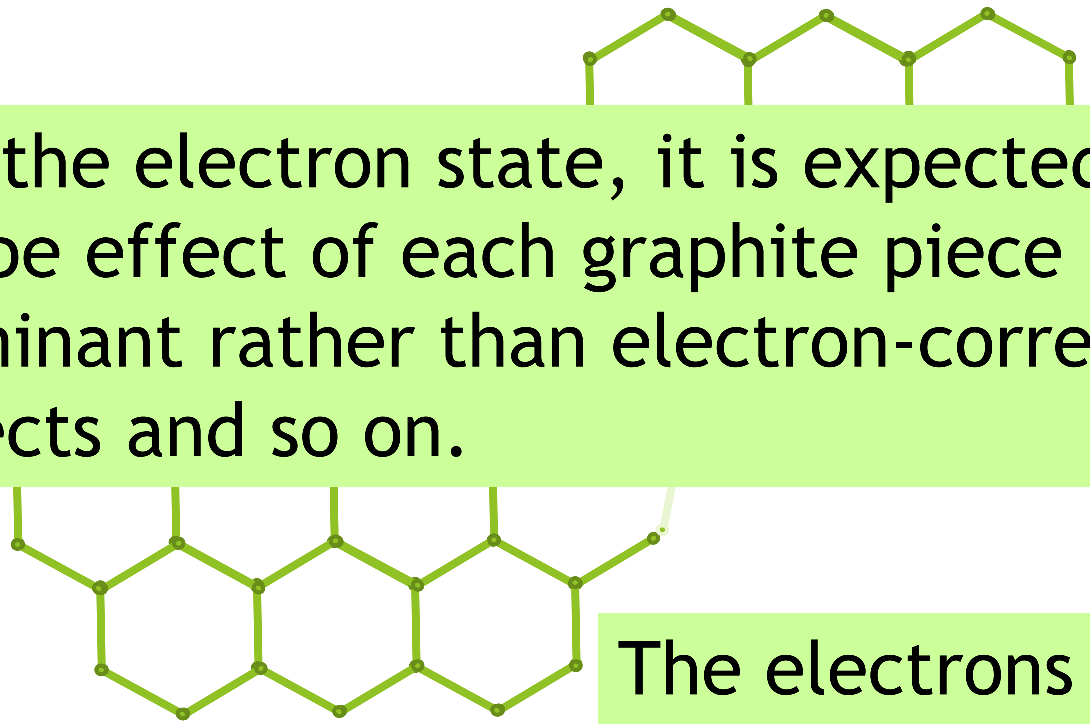
The electrons are localized there!

At the connection of the graphite pieces, the phonons are easily excited and coherency of the electron wave function is lost there



The electrons are localized in each graphite piece.

At the connection of the graphite pieces, the phonons are easily excited and coherency of the electron wave function is lost there



For the electron state, it is expected that the shape effect of each graphite piece must be dominant rather than electron-correlation effects and so on.

The electrons are localized in each graphite piece.

Tight binding approximation of (one-body) Hamiltonian is given by

$$H = \varepsilon_0 D_{\mathcal{G}} - \gamma_0 A_{\mathcal{G}},$$

γ_0=3.16[eV]

The eigenvalues of Hamiltonian is given by the eigenvalues of the adjacency matrix.

This approach might be justified by the full statistical and quantum theory

$$Z=\int D\psi D\psi D\varphi D\mathbf{p} D\mathbf{q} \cdot \exp(S_0)$$

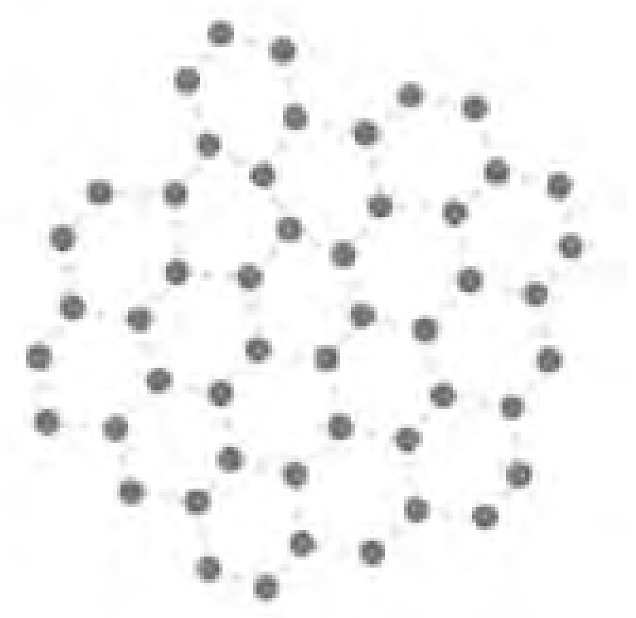
$$S_0[\psi_e, \bar{\psi}_e, \mathbf{p}_l, \mathbf{q}_l, \varphi] = -(8\pi)^{-1} \int d\tau\, d^3x (\nabla\varphi(\mathbf{x},\tau))^2$$

$$+ \int d\tau\, d^3x \sum_s \bar{\psi}_{es}(\mathbf{x},\tau)(\partial_\tau + (2m)^{-1}\nabla^2 + \lambda_e + ie\varphi(\mathbf{x},\tau))\psi_{es}(\mathbf{x},\tau)$$

$$+ \int d\tau \sum_l [i\mathbf{p}_l(\tau)\partial_\tau \mathbf{q}_l(\tau) - (2M)^{-1}\mathbf{p}_l^2(\tau) + \lambda_i - ie\varphi(\mathbf{q}_l(\tau),\tau)].$$

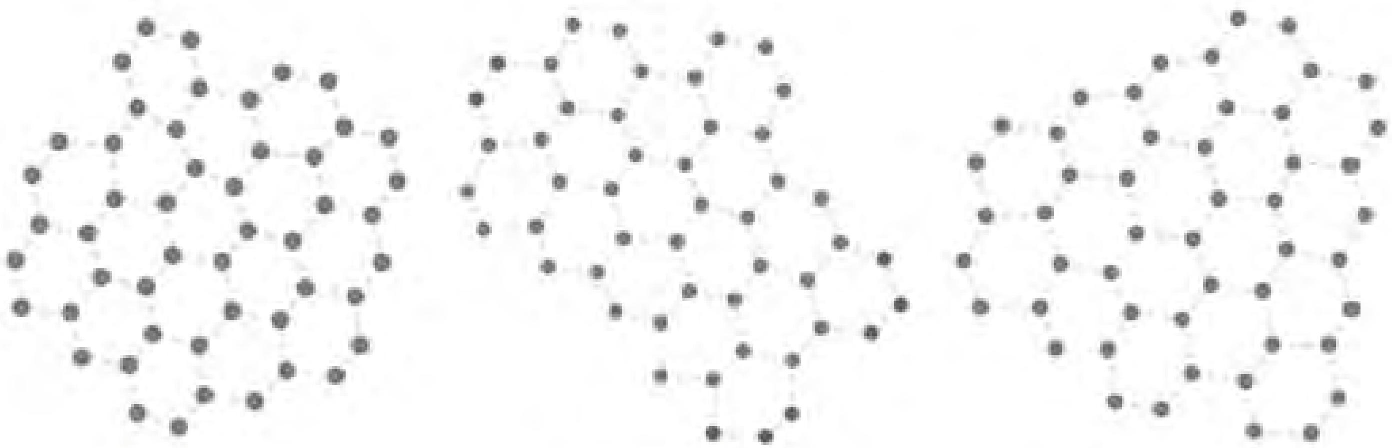
V. N. Popov,
Functional integrals and collective excitations

Using a software of graph theory (Graphtea), let us consider a piece of grpahite.



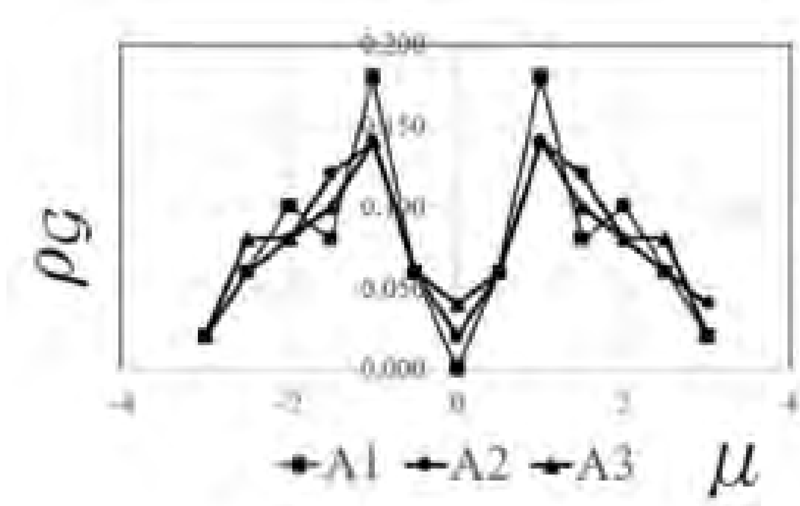
(A1):50

Using a software of graph theory (Graphtea), let us consider pieces of grpahite.

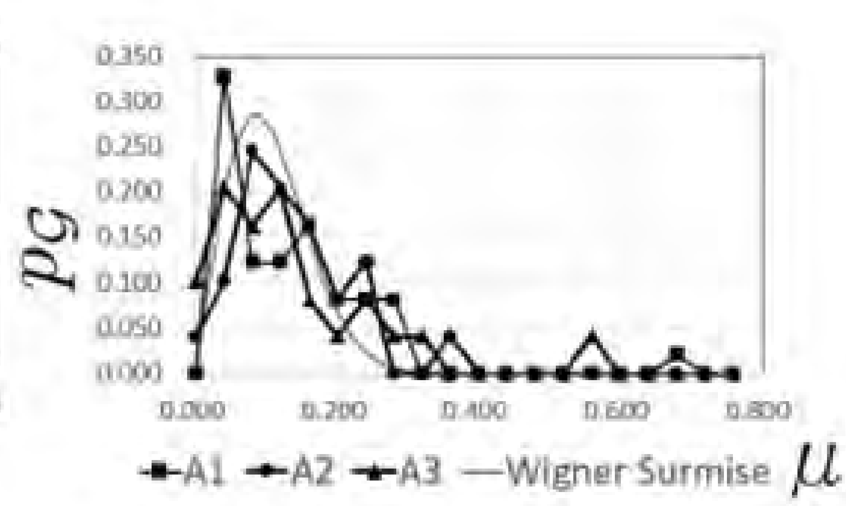


(A1):50 (A2):50 (A3):50

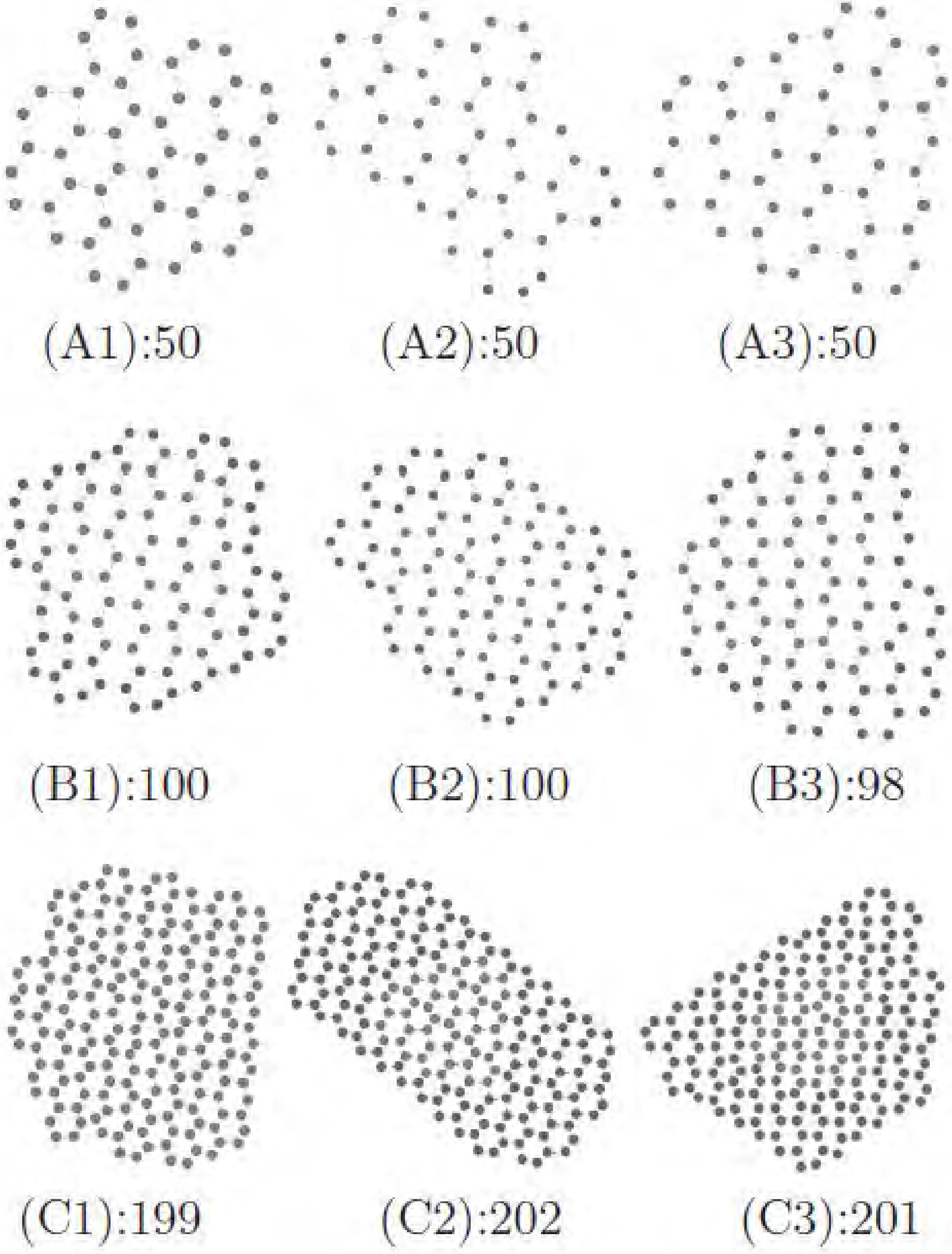
Using a software of graph theory (Graphtea), let us consider pieces of graphite and compute the eigenvalue of the adjacent matrix

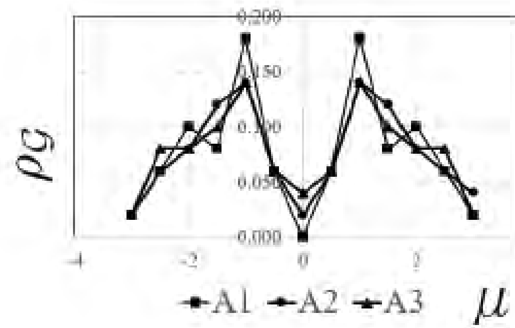


The distribution of eigevalues



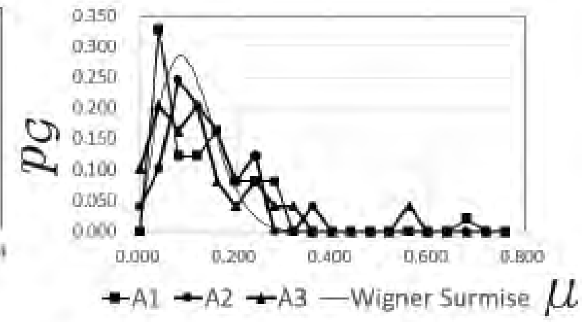
The distribution of spacings


(A1):50
(A2):50
(A3):50
(B1):100
(B2):100
(B3):98
(C1):199
(C2):202
(C3):201



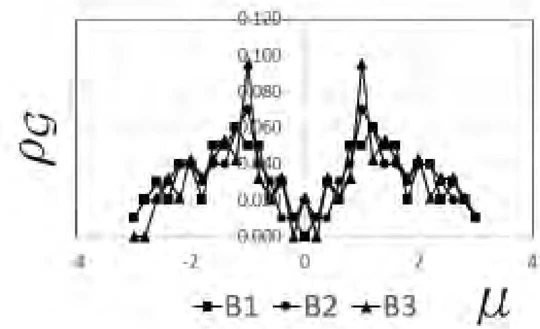
ρg
A1 A2 A3
μ

(a)



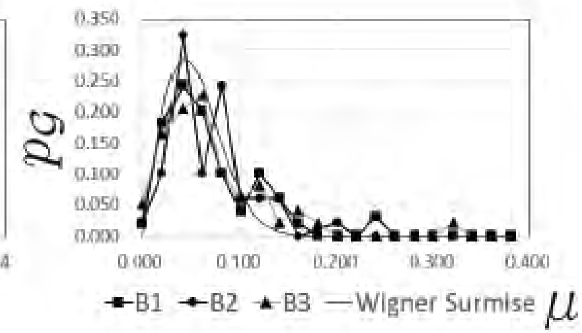
ρg
A1 A2 A3 Wigner Surmise
μ

(b)



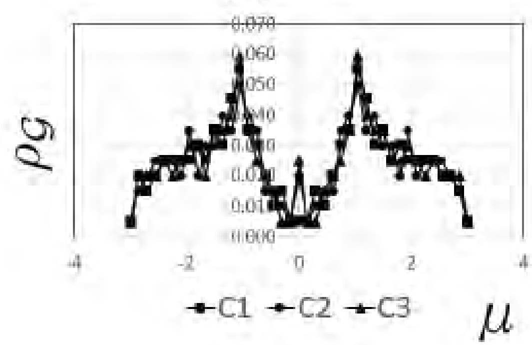
ρg
B1 B2 B3
μ

(c)



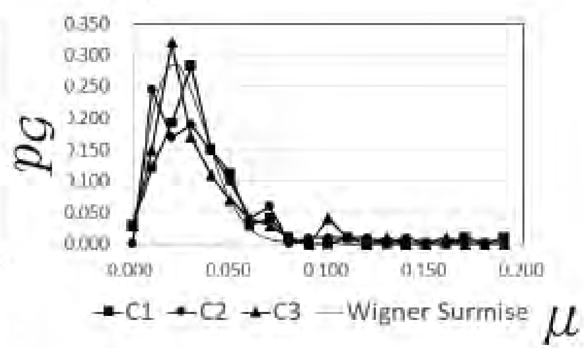
ρg
B1 B2 B3 Wigner Surmise
μ

(d)



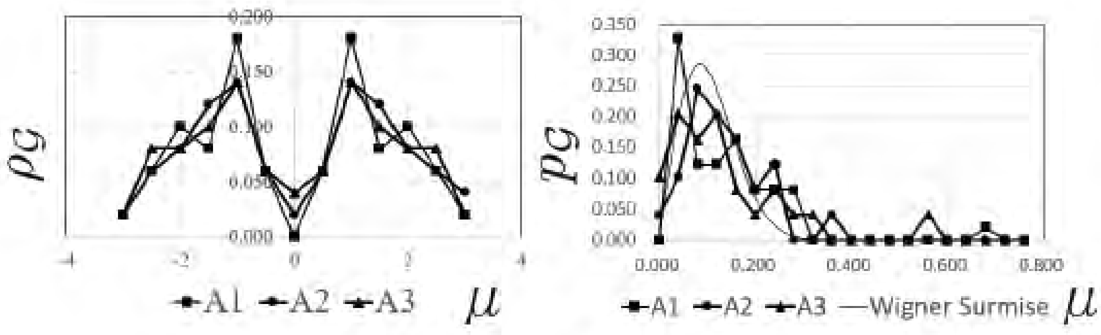
ρg
C1 C2 C3
μ

(e)



ρg
C1 C2 C3 Wigner Surmise
μ

(f)

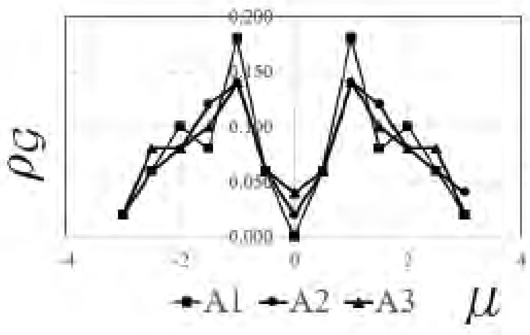


(a) (b)

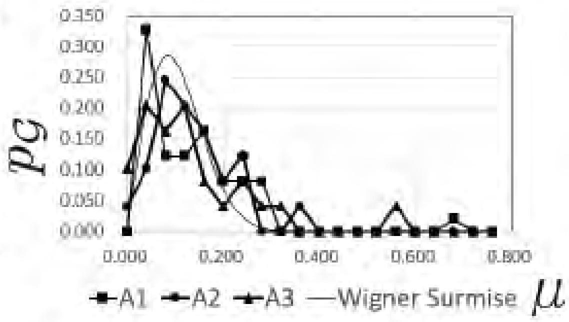
- The eigenvalue distribution asymptotically approaches to that of graphite
- The spacings are approximated well by the Wigner surmise

$$p_G = 2 \frac{\mu}{\alpha^2} \exp(-\mu^2/\alpha)\, d\mu$$

(e) (f)



(a)



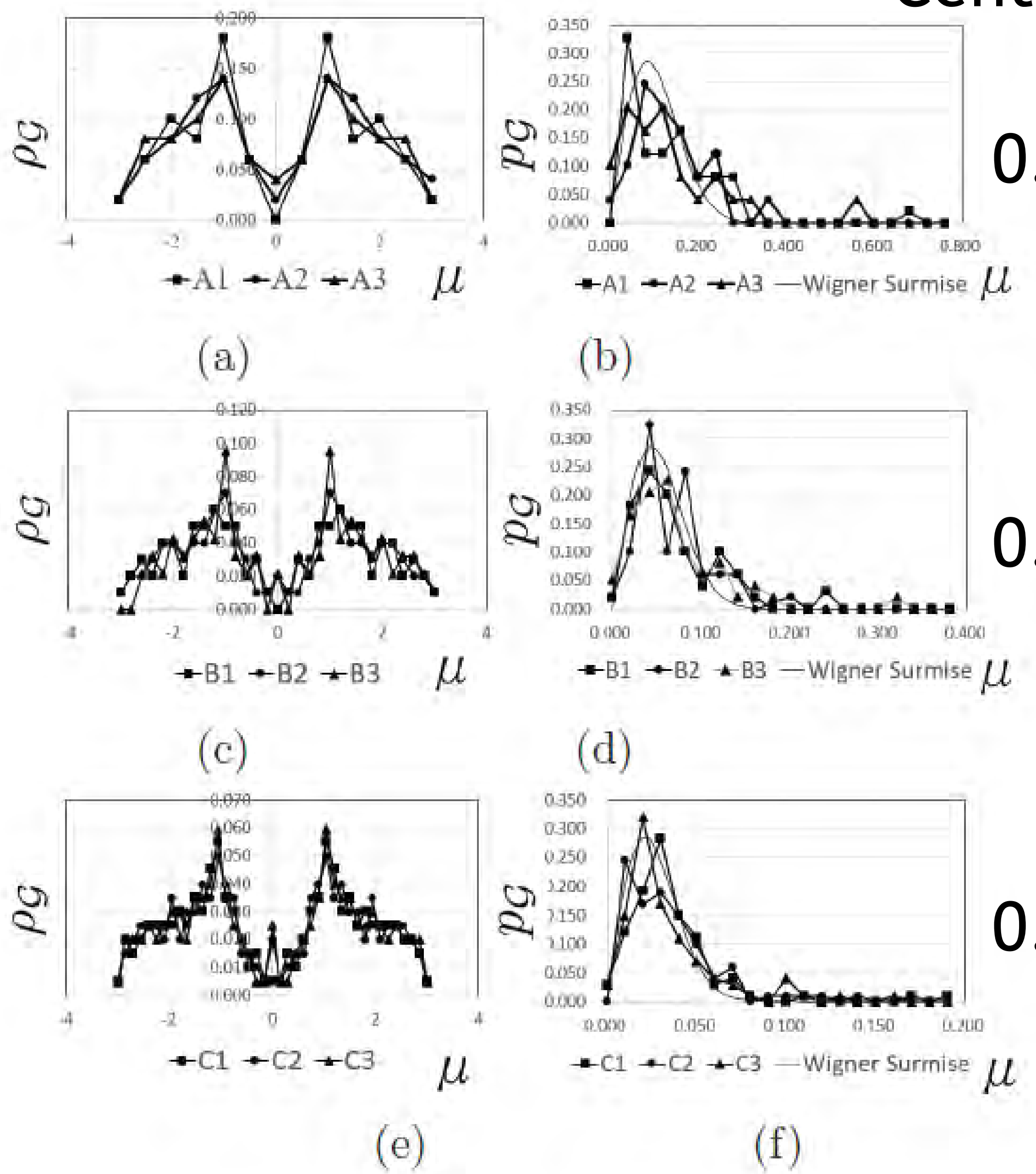
(b)

- The eigenvalue distribution asymptotically approaches to that of graphite
- The spacings are approximated well by the Wigner surmise. Its center gravity is

$$2\int \frac{\mu^2}{\alpha^2}\exp(-\mu^2/\alpha)\, d\mu = \frac{\sqrt{\pi}}{2}\alpha$$

(e) (f)

Center of gravity



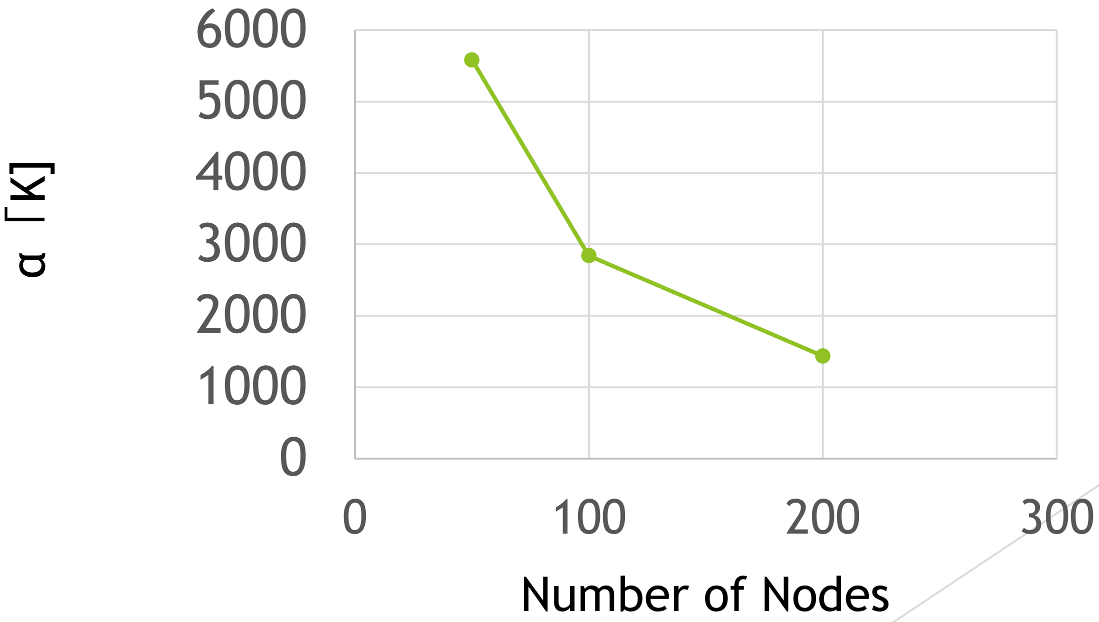
0.135

0.069

0.035

Table 1. Center of gravity of $p_{\mathcal{G}}$ and α [K]

	1	2	3	average	α [K]
A-type	0.136	0.135	0.134	0.135	5586
B-type	0.069	0.068	0.069	0.069	2842
C-type	0.035	0.035	0.034	0.035	1436



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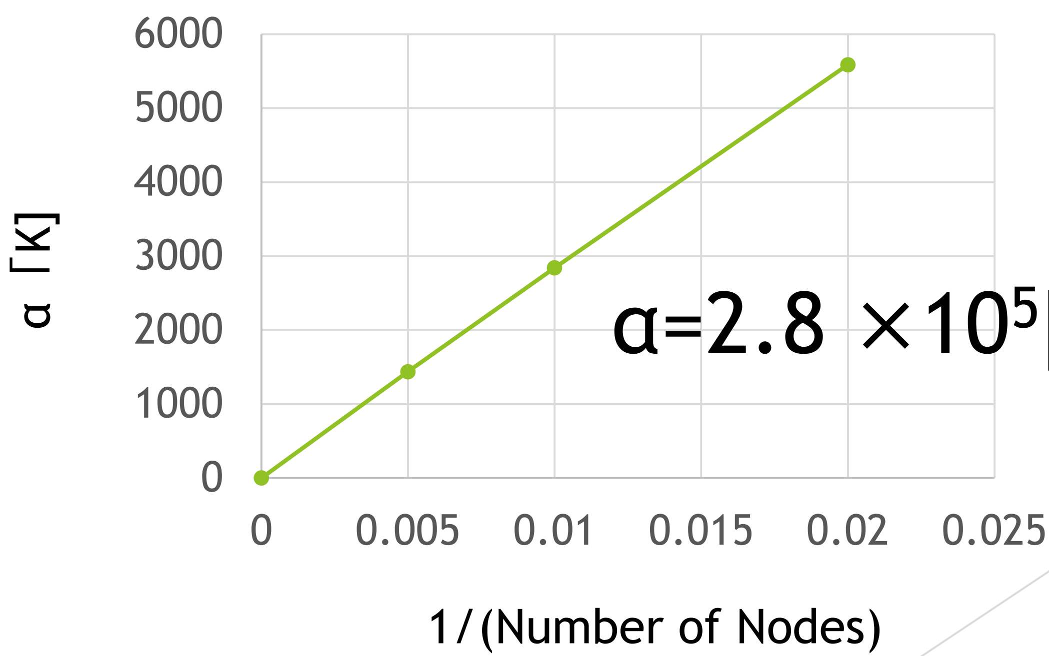
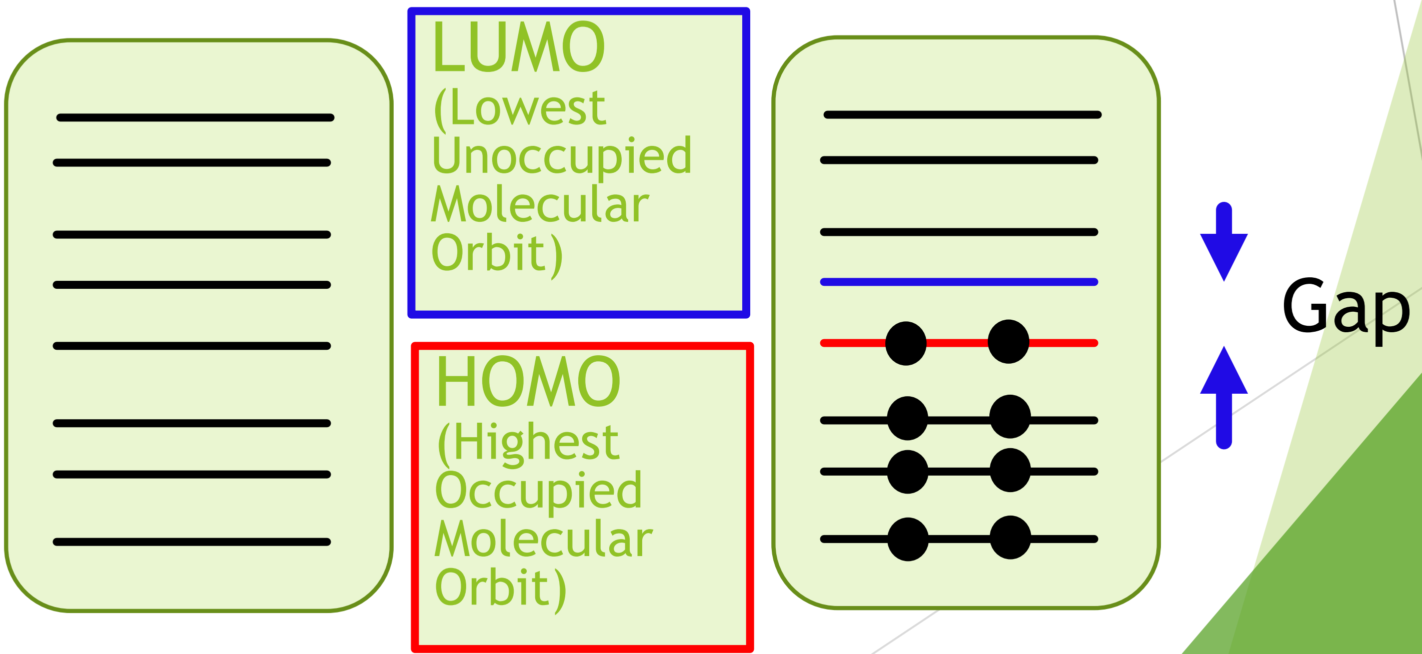


$\alpha = 2.8 \times 10^5 [\mathrm{K}] / N$

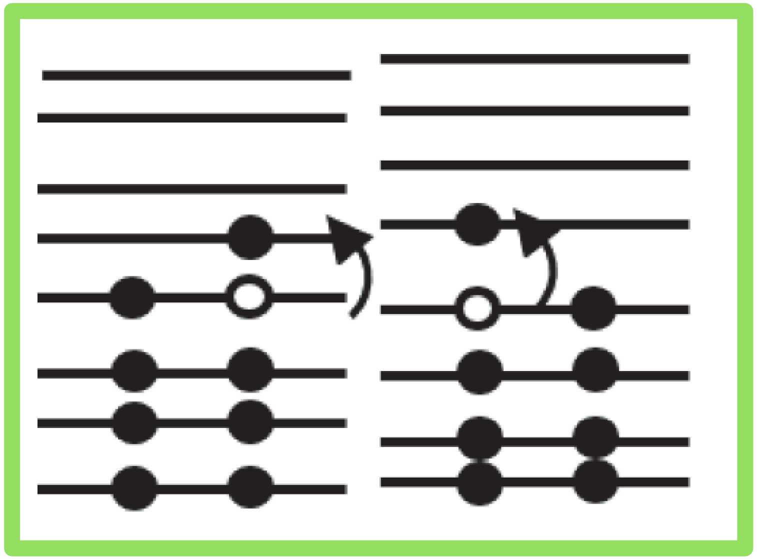
Due to spin effect, the half of the eigenvalues is filled and we are concerned with the gap of the HOMO and the LUMO.

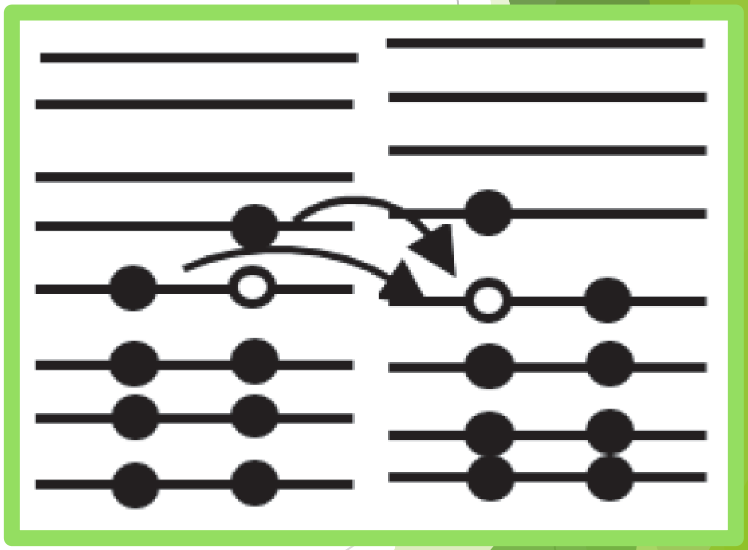
Let us assume that the gap may also obey the Wigner surmise.



Picture of the activation hopping

The phonon (of the lattice temperature) assists the hopping





Picture of the activation hopping

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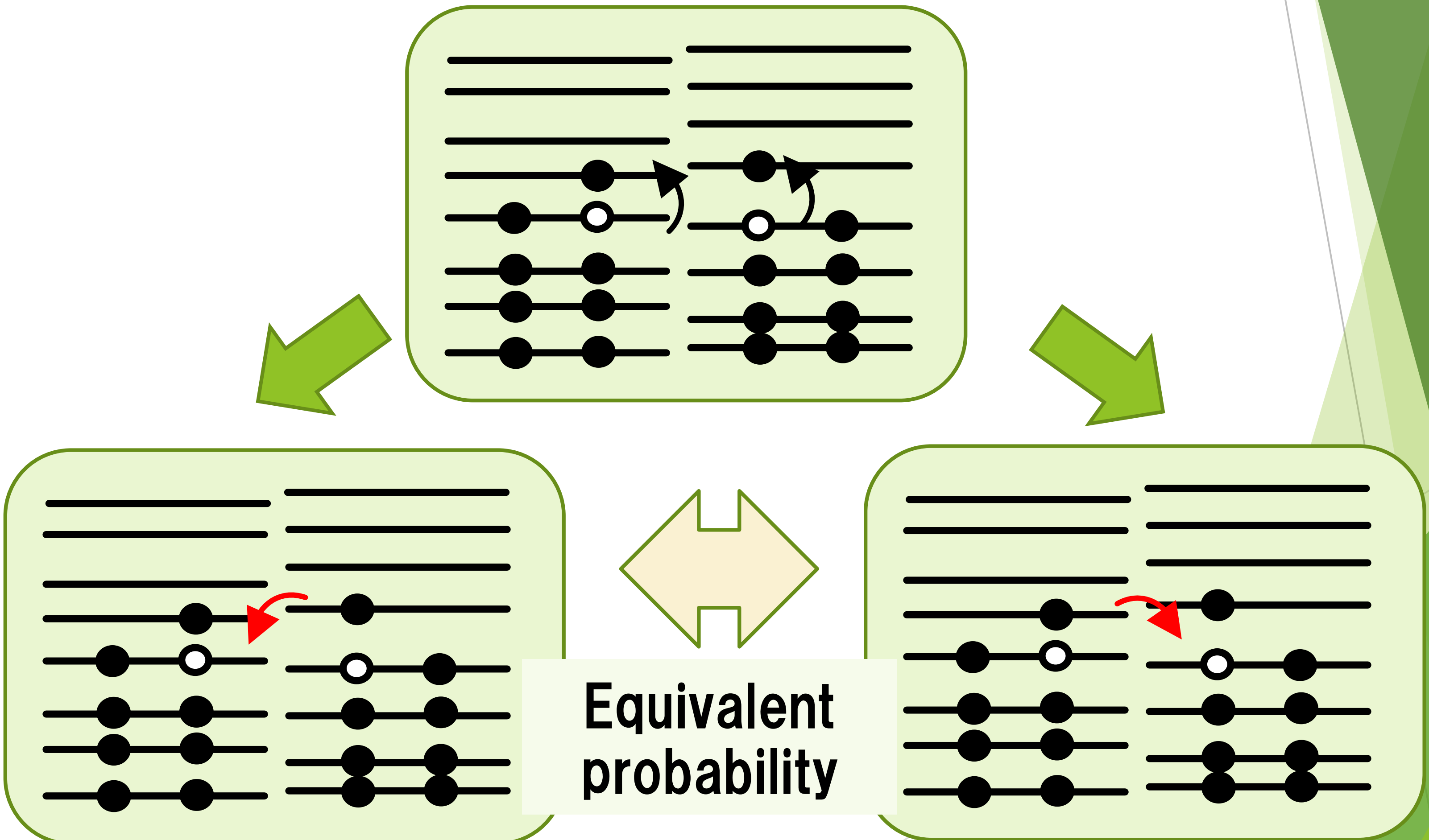

Picture of the activation hopping

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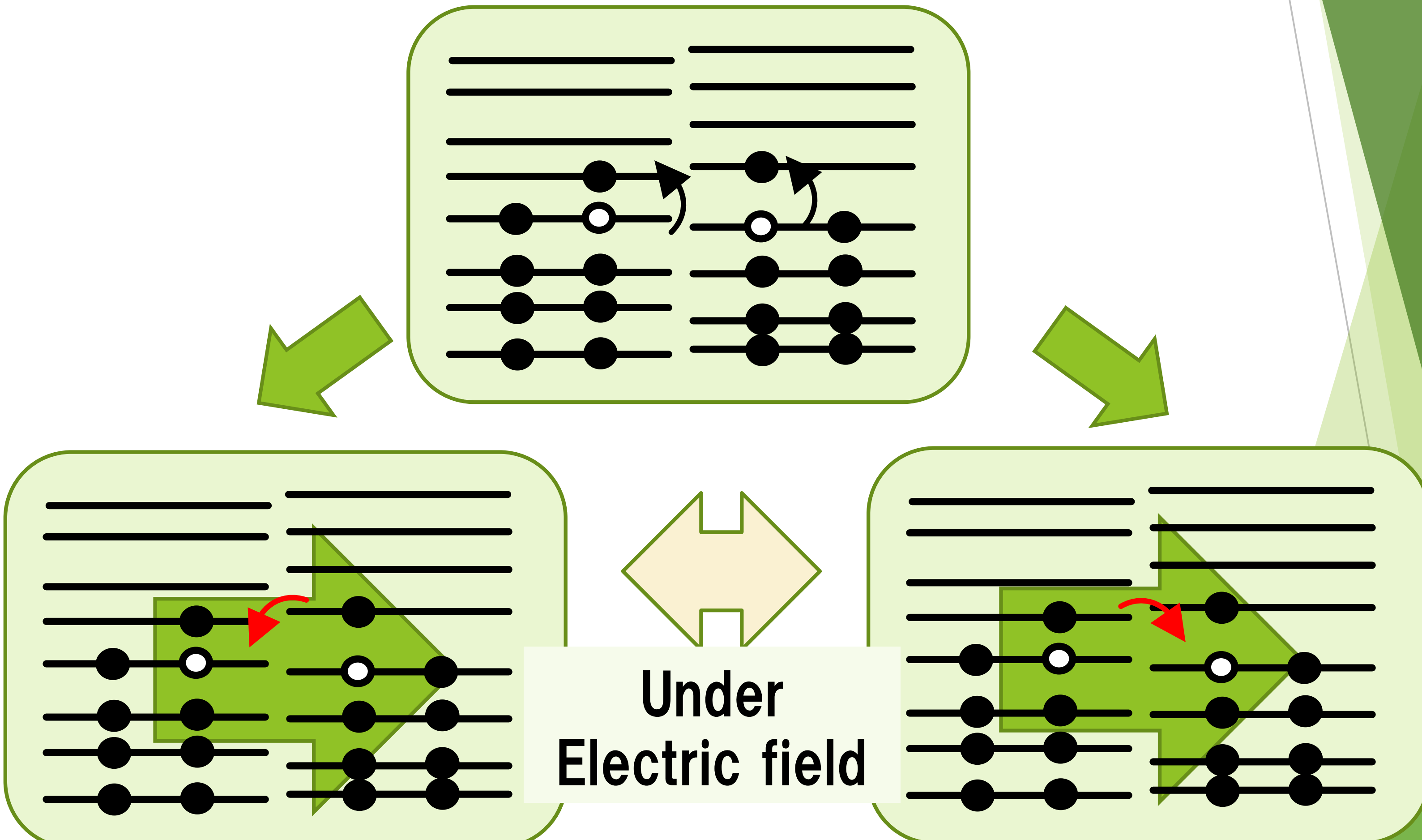

Picture of the activation hopping

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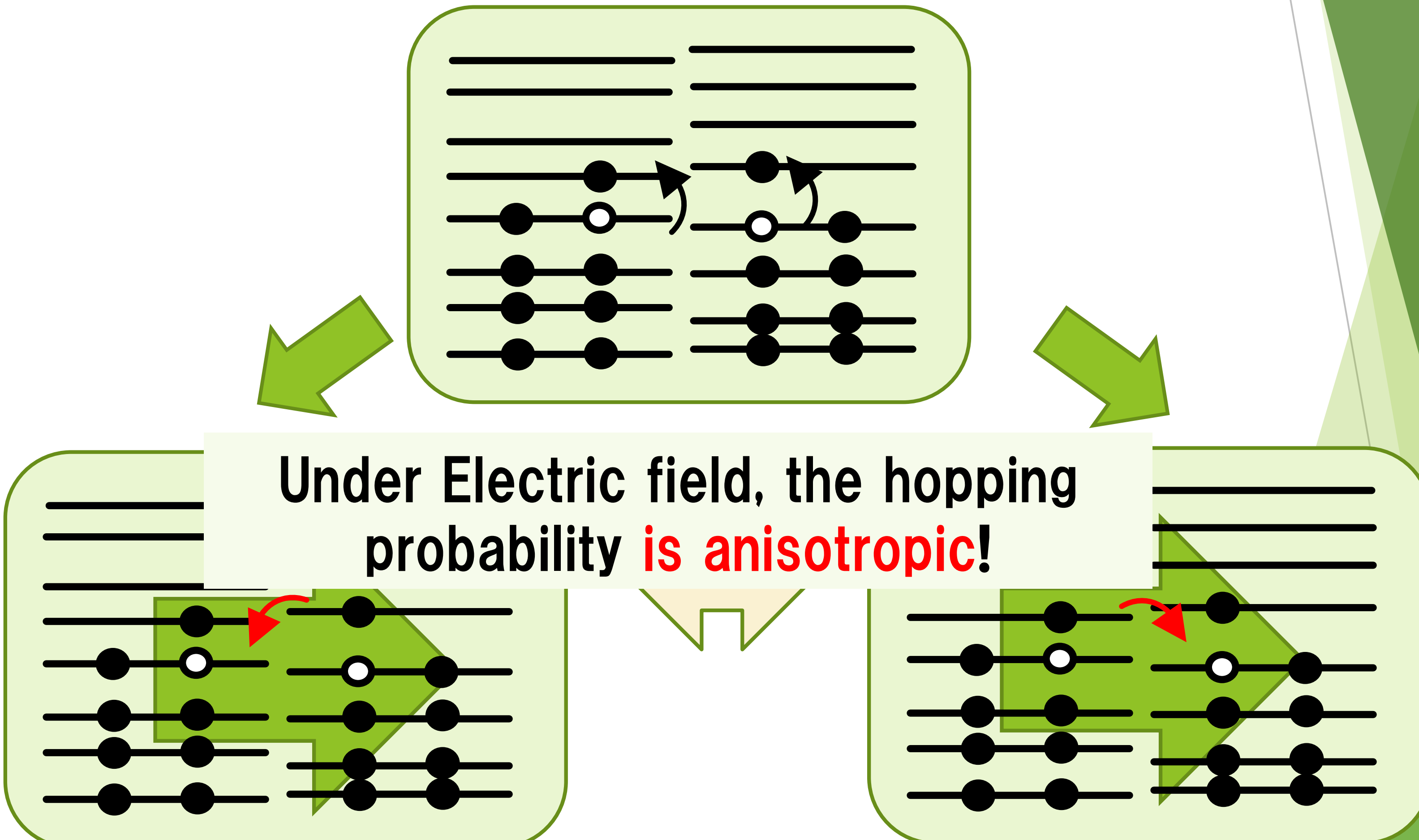
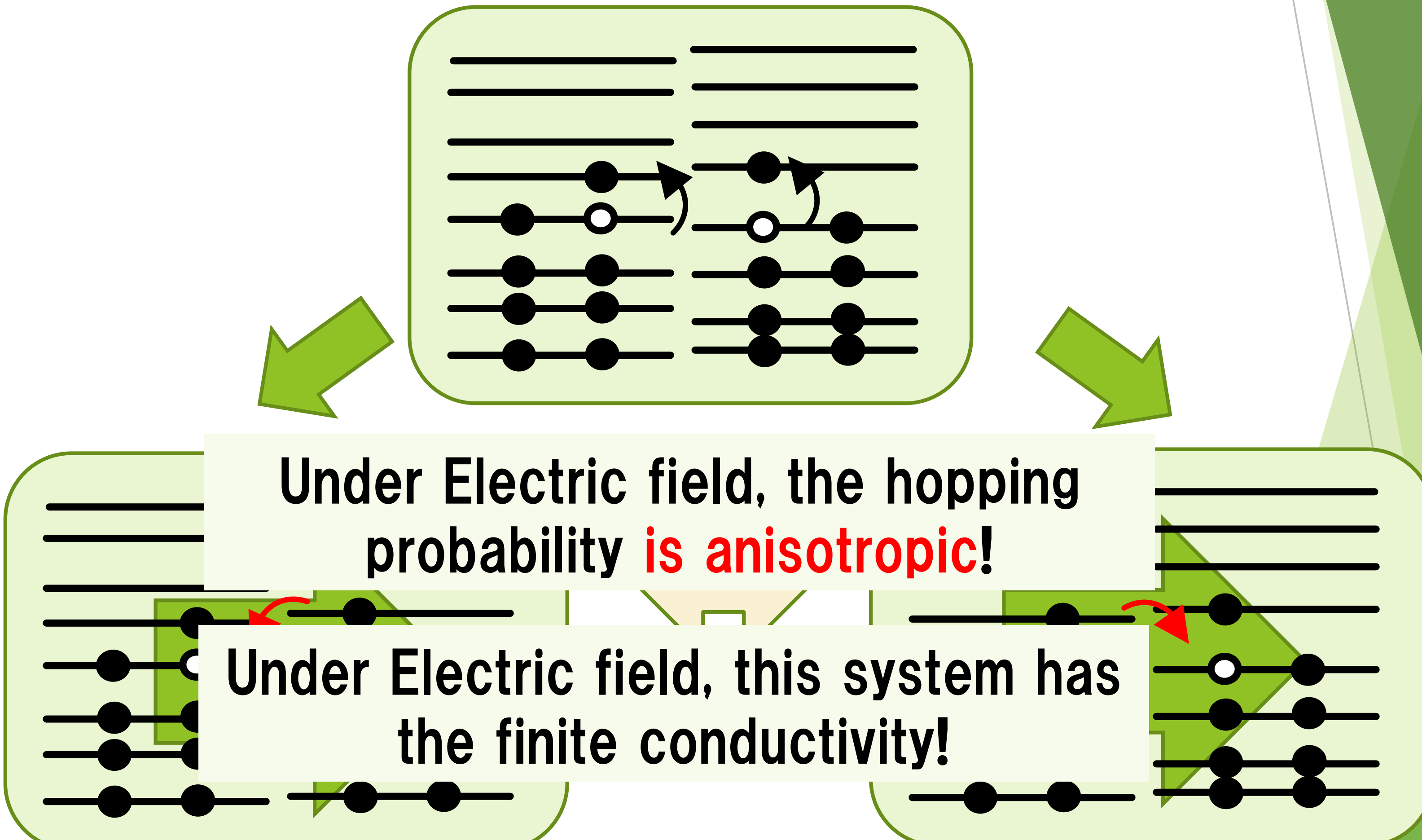


Under Electric field, the hopping probability is anisotropic!

Picture of the activation hopping

The phonon (of the lattice temperature) assists the hopping



Under Electric field, the hopping probability is anisotropic!

Under Electric field, this system has the finite conductivity!

Picture of the activation hopping

The phonon (of the lattice temperature) assists the hopping and causes the local conductivity under the electric field!

$$\sigma \propto e^{-\delta E/T}$$

δE obeys the Wigner surmise

$$p_G = 2\frac{\mu}{\alpha^2}\exp(-\mu^2/\alpha)\,d\mu$$

Picture of conductivity in randomized material.

1. There are conductive paths C_i, (i=1,2,⋯,n) .
2. In each path C_i which consists of local conductors σ_{ij}, the conductivity of each path is given as

$$\sigma_i = \left(\sum_j \frac{1}{\sigma_{ij}} \right)^{-1}$$

In each path, there is highly resistant place (gap)

$$\sigma_i \fallingdotseq \left(\max_j \frac{1}{\sigma_{ij}} \right)^{-1}$$

3. The total conductivity is given by,

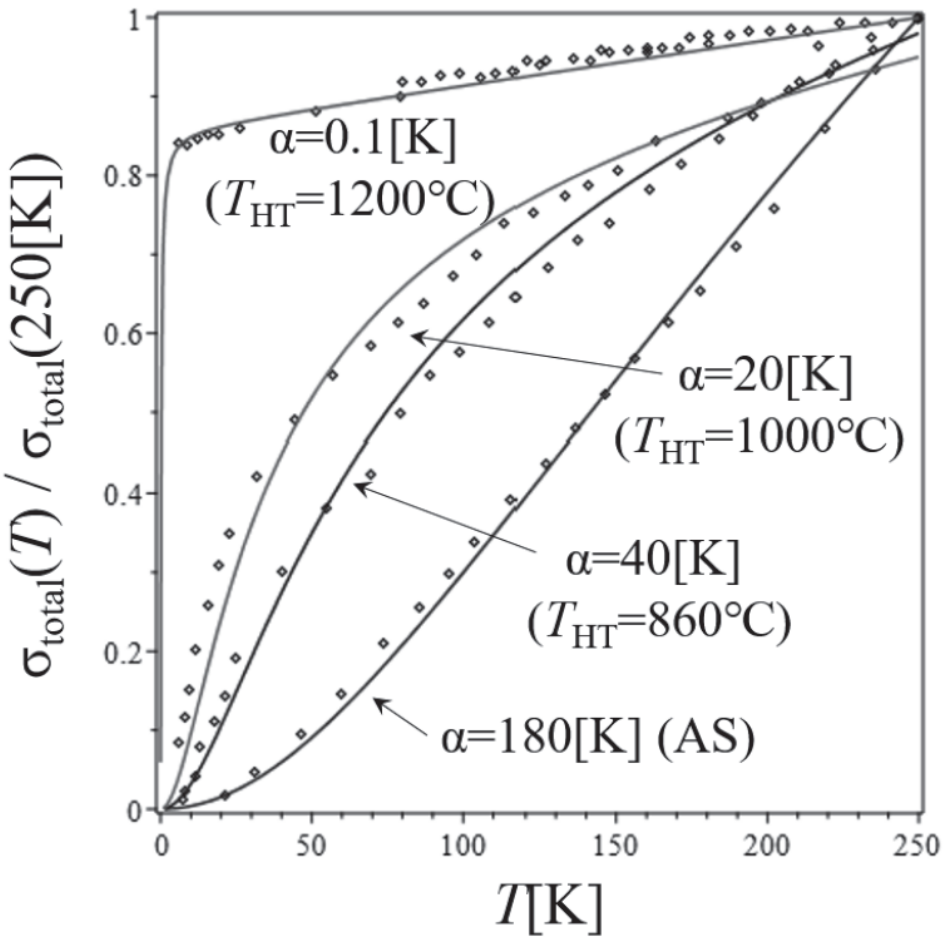
$$\sigma_{total} \fallingdotseq \sum_i \min_j \sigma_{ij}$$

Sum over the local conductivity

$$\begin{aligned}\sigma_{\text{total}}(T) &= \sigma_0(T)\int dE_{\text{G}}\, p_{\text{WS}}(E_{\text{G}})\exp(-E_{\text{G}}/k_{\text{B}}T) \\ &= \sigma_0(T)\left[1-\sqrt{\pi}\frac{\alpha}{T}\exp\left(\frac{\alpha^2}{T^2}\right)\cdot \text{erfc}\left(\frac{\alpha}{2T}\right)\right],\end{aligned}$$

Fitting parameters:

- α
- ε : $\sigma_0(T)=\sigma_0(1+\varepsilon \cdot T)$



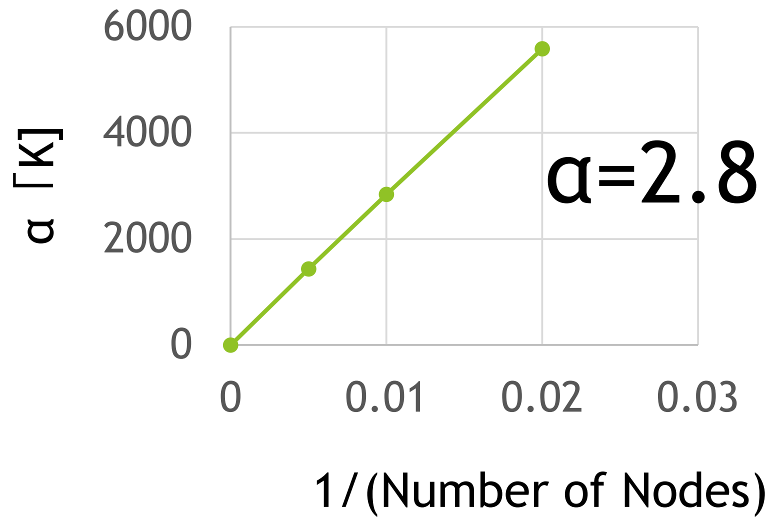
Fitting parameters:

- α
- ε : $\sigma_0(T)=\sigma_0 \cdot (1+\varepsilon \cdot T)$, $\varepsilon=1.5\times10^{3}[K^{-1}]$

α : parameter

Table 2. α and R of the ACF

		α [K]	N	R[nm]
case I	AS	180	1.6×10^3	3.2
case II	$T_{HT} = 860$[°C]	40	7.0×10^3	6.7
case III	$T_{HT} = 1000$[°C]	20	1.4×10^4	9.5
case IV	$T_{HT} = 1200$[°C]	0.1	2.8×10^6	1.3×10^2

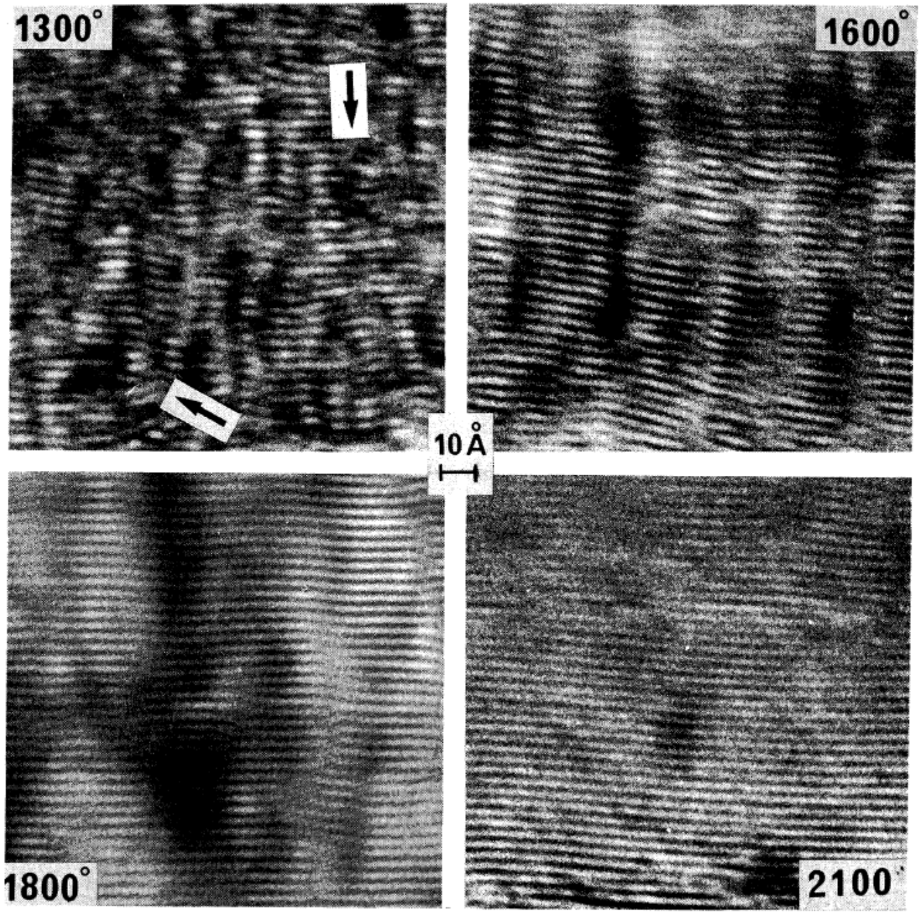


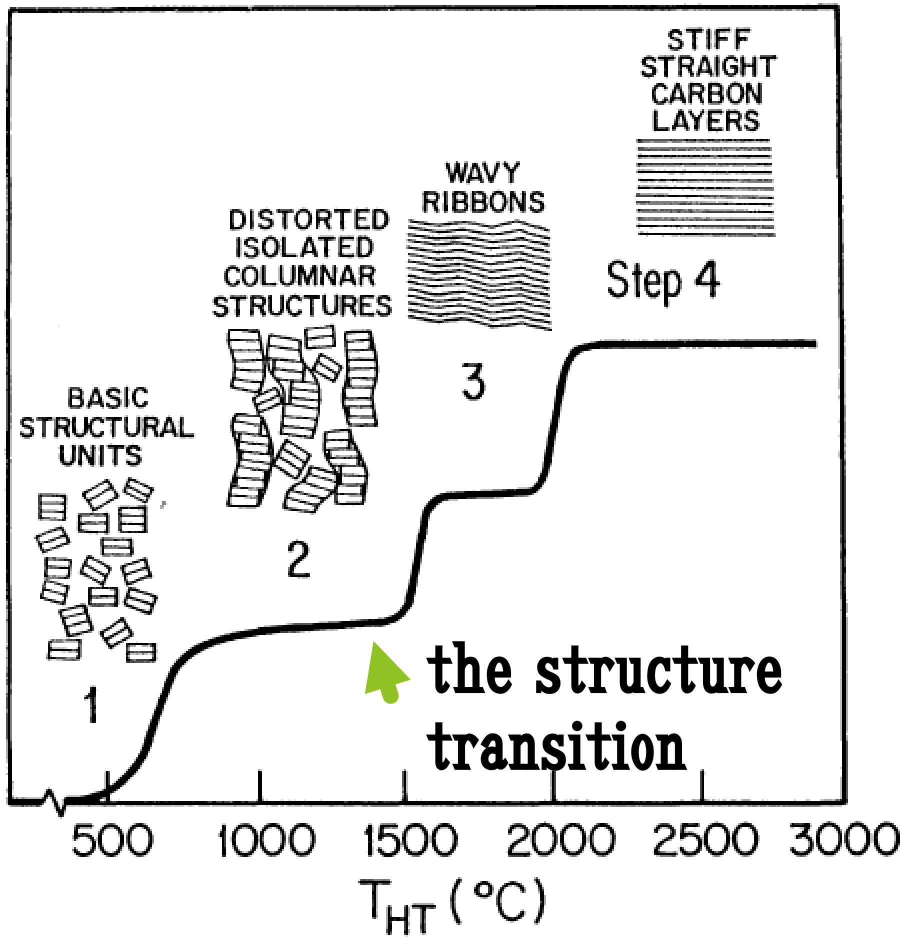
$\alpha = 2.8 \times 10^5[K]/N$

$N = \pi * R^2 / a^2$ 2-dim

$a = 0.142$[nm],

Heat treatment makes the structure changes
The higher temperature is, the size of pieces is larger!





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In Case I, if we consider the three sheets and then the size agrees with Kuriyama's investigation La ∼ 3 nm.
From Case I to case III, the size become bigger.

α : parameter

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case IV	$T_{HT} = 1200[°C]$	0.1	2.8×10^6	1.3×10^2

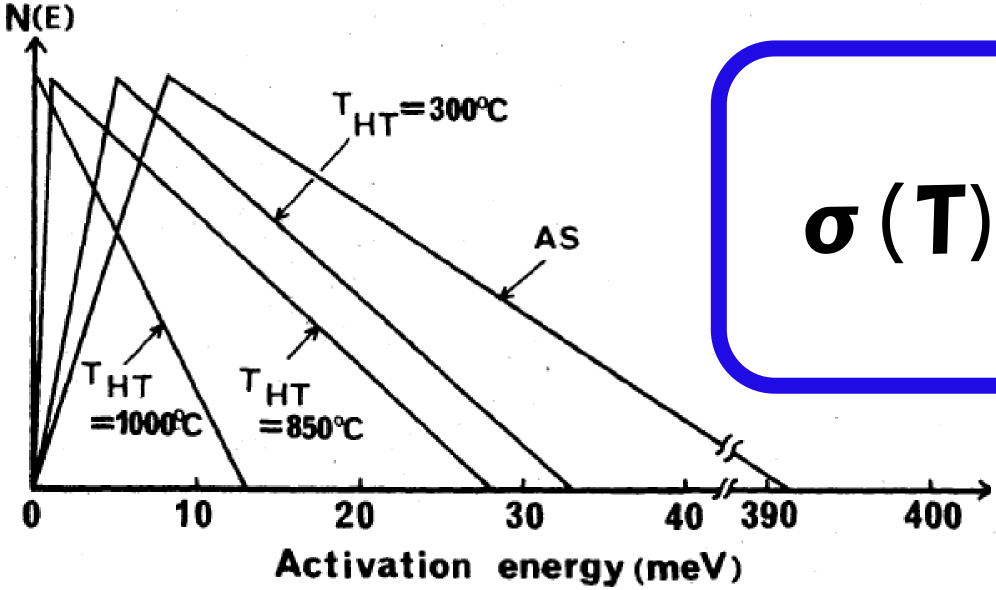
In Case IV, we may consider three dimensional effect. Due to it, we could interpret that the metal-insulator transition occurs.

→ metal-insulator transition may be related to the structure transition though Laman finds the the difference!

Advanced Mathematical Investigation for conductivity of highly disordered carbon systems; percolation and graph zeta function

1. Activation carbon fiber
2. Conductivity of ACFs
   Kuriyama's Investigation
3. Conductivity of percolation
4. Graph Theory
5. New proposals on the conductivity
6. **Summary**

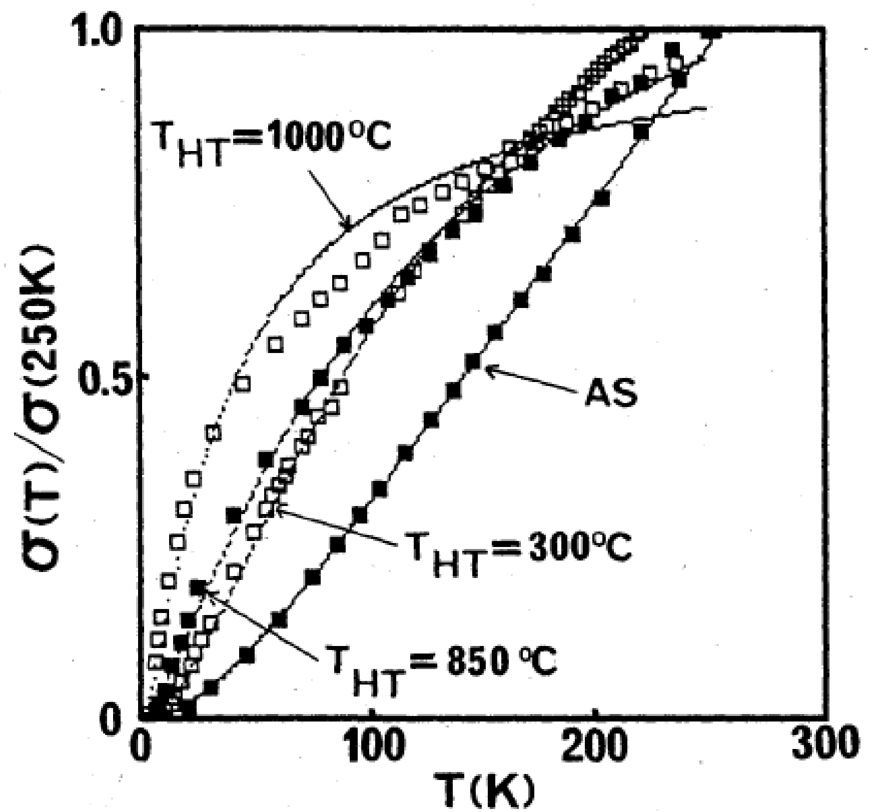
Kuriyama 's proposal (Phys.Rev.B 1993)



$$\sigma(T) = \sigma_0 \int N(E) e^{-E/T} dE$$

Kuriyama 's proposal recovers his experimental results

Phys.Rev.B 1993



Summary of this work

Purposes of this Study are

1) To give the microscopic origin of the Kuriyama’s mechanism, and
2) To improve his results

Summary of this work

Purposes of this Study are

1) To give the microscopic origin of the Kuriyama's mechanism, and
2) To improve his results

1) We give its microscopic origin based on graph theory and percolation theory.
2) Our model enables us to deal with the metallic region and gives the relation between the metal-insulator transition and structure transition.

Summary of this talk

I gave an example of advanced mathematical investigation.
The day, that only applied mathematics plays role of the application to indusity and science, finishs.

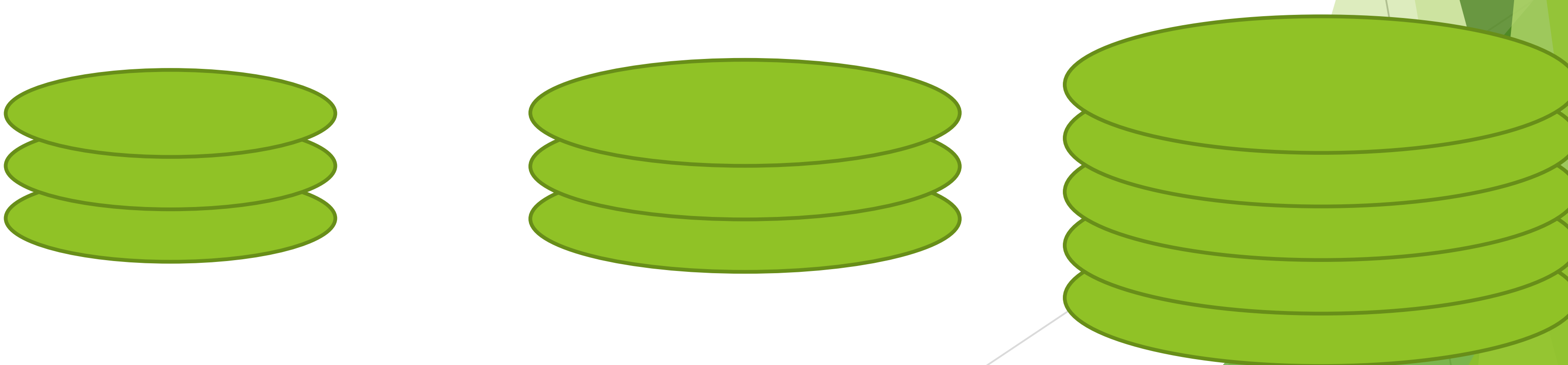
Crucial Industrial problems basically cannot be solved in the single field of mathematics or in the single field in science in general.

I hope that researchers of various fields of mathematics and science collaborate to solve the crucial problems.

Thank you!

α : parameter

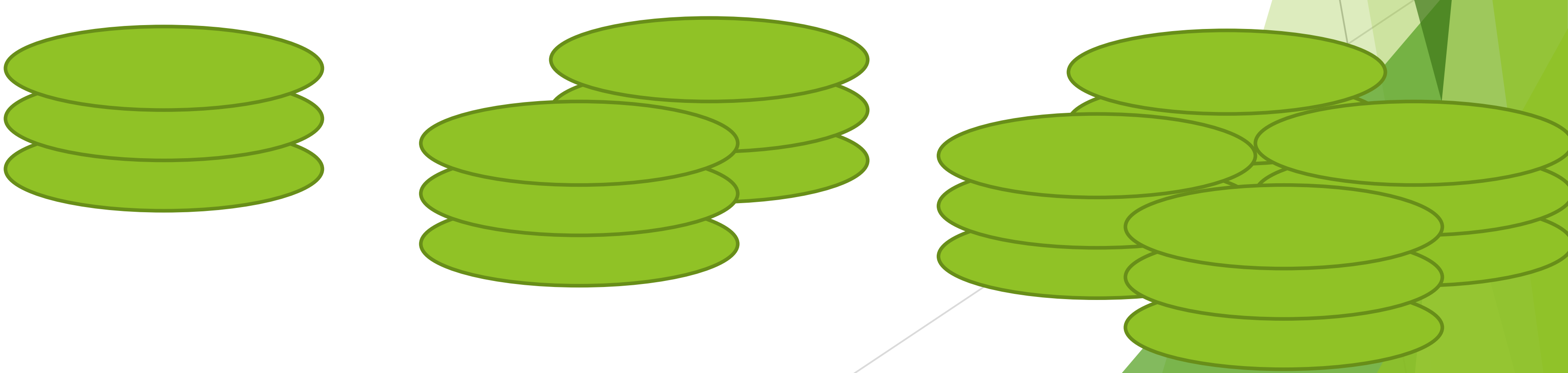
		N	sheats	N/sheat	R
case I	AS	1.60E+03	3	5.33E+02	1.85E+00
case II	T_{HT}=860[℃]	7.00E+03	3	2.33E+03	3.87E+00
case III	T_{HT}=1000[℃]	1.40E+04	3	4.67E+03	5.47E+00
case IV	T_{HT}=1200[℃]	2.80E+06	30	9.33E+04	2.45E+01



α : parameter

		N	N/N_{AS}
case I	AS	1.60E+03	1.0
case II	T_{HT}=860[℃]	7.00E+03	4.4
case III	T_{HT}=1000[℃]	1.40E+04	8.8
case IV	T_{HT}=1200[℃]	2.80E+06	1750.0

Connections of granule increases



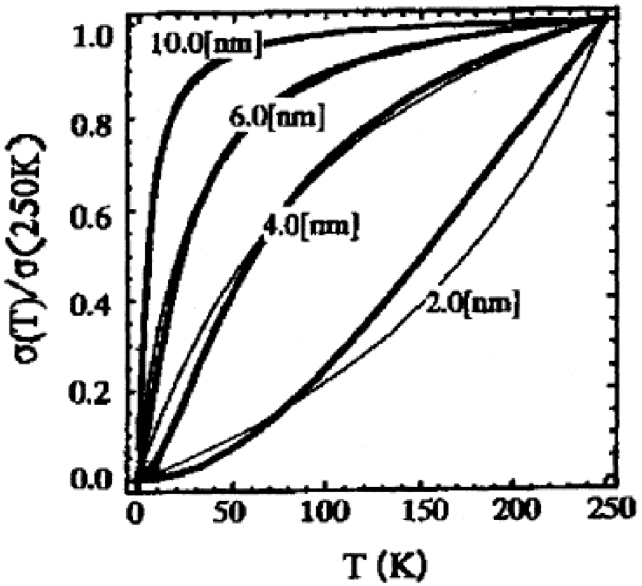


FIG. 2. The temperature dependence of normalized conductivity for σ^{Wigner} and $\sigma^{\mathrm{Poisson}}$. Thin lines correspond to the curves for $\sigma^{\mathrm{Poisson}}$ and thick ones are those for σ^{Wigner}. They are parametrized by ξ or R, respectively. ξ and R are taken for 2 nm, 4 nm, 6 nm, and 10 nm.

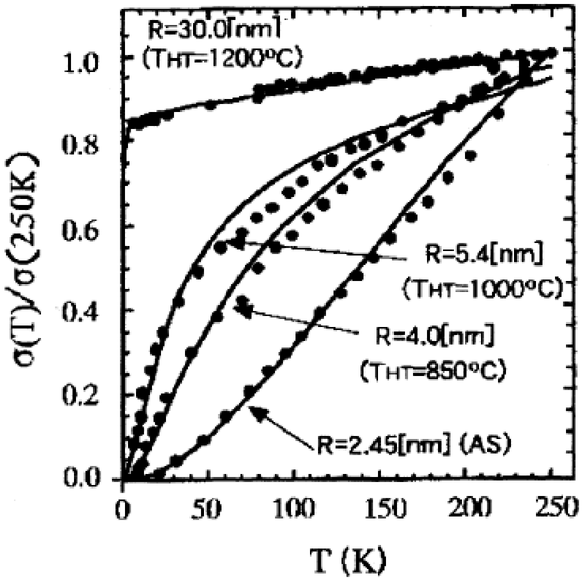


FIG. 3. The temperature dependence of normalized conductivity. The dotted points show the experimental results of the temperature dependence on normalized conductivity $\sigma(T)/\sigma(250\ \mathrm{K})$ for as-prepared (AS) and heat-treated ACF's (Fig. 1 in Ref. 5). Fits of these points are given by curves, whose fitting parameters R and ϵ in Eq. (17) are $R=2.45$ nm, 4.00 nm, 5.40 nm, 30.0 nm, and $\epsilon^{-1} = 1.5\times10^3\ \mathrm{K}^{-1}$.

M. Suzuki PRB 2000

Three main philosophers for the advanced mathematical investigation

Ferdinand de Saussure
(1857-1913)

Thomas Samuel Kuhn
(1922-1996)

Edmund G. A. Husserl
(1859-1938)

Three main philosophers for the advanced mathematical investigation

Ferdinand de Saussure
(1857-1913)

"A representing tool and object represented are not divisible" in Semiotics

⇔Without language, we can not describe a concept!

⇒If one controls something, he should express it using a language.

Language of technology is Mathematics

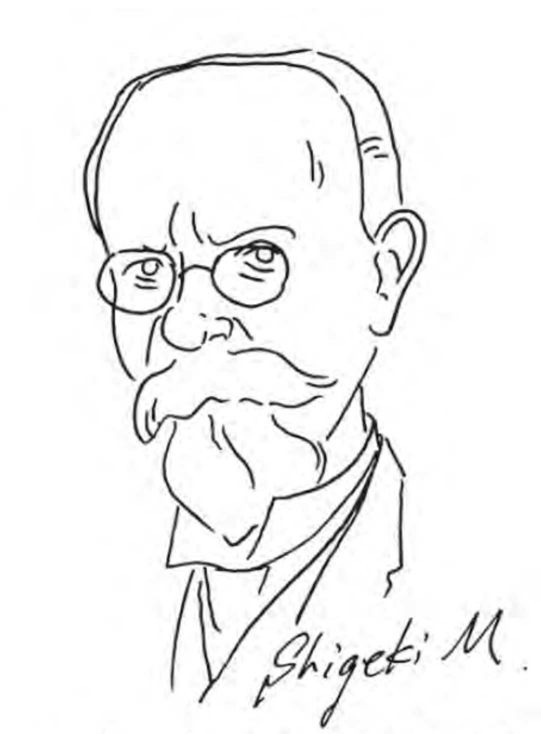
（We control a technology in terms of Mathematics）

Thomas Samuel Kuhn
(1922-1996)

The interconnection among paradigms is impossible whereas the paradigm is the base of science.

⇒ To overcome this impossibility is key for the new technology.

⇒ Mathematics has a potential to overcome it.



Edmund G. A. Husserl
(1859-1938)

Mathematics is the limit-operation of the language in technology.

⇒When we apply mathematics to real world, we should recall the limit.

⇒ We should notice the difference between mathematical world and real world.